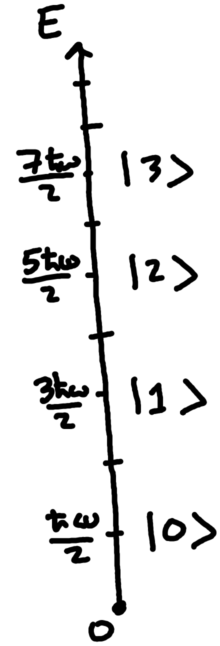
Quantum Harmonic Oscillator Cheat Sheet

$$H = \frac{\hat{P}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2$$



Energy eigenstates:

$|n\rangle$ has energy $\hbar\omega(n+\frac{1}{2})$

Creation operator: $a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} - \frac{i}{\sqrt{2m\omega\hbar}}\hat{p}$

Annihilation operator: $a = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} + \frac{i}{\sqrt{2m\omega\hbar}}\hat{p}$

$$[\hat{x},\hat{p}] = i\hbar \Rightarrow [a,a^\dagger] = \mathbb{1} \quad [H,a] = -\omega a \quad [H,a^\dagger] = \omega a^\dagger$$

these imply

"raising operator"

$$a^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$a\,|n\rangle = \sqrt{n}\,|n-1\rangle$$

"lowering operator"

$$\Rightarrow a^\dagger a |n\rangle = n|n\rangle$$

"number operator"

Use these for most calculations via:

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(a+a^\dagger) \quad \hat{p} = -i\sqrt{\frac{\hbar m\omega}{2}}(a-a^\dagger) \quad H = \hbar\omega(a^\dagger a + \tfrac{1}{2})$$

2D oscillator: same as 2 independent 1D oscillators: define $a_x, a_x^\dagger, a_y, a_y^\dagger$, states are $|n_x, n_y\rangle$, etc...