Consider states

$$|n\ l\ m\rangle \otimes |s_z\rangle$$

of a hydrogen atom, with $l=1$ and $n=2$.

We'll denote these by $|m\rangle \otimes |s_z\rangle$.

a) Write each of the possible basis states, and for each state, write the eigenvalue of

$$J_z = L_z + S_z$$

b) These states aren't eigenstates of total angular momentum J^2. But some linear combinations of them are eigenstates of J^2 and J_z (label these by $|J\ M\rangle$). Based on the values of J_z you found, and the fact that states with some J always come in groups with $M = -J, -J+1, \dots, J$, what are the possible values of (J, M)?

c) Each $|J\ M\rangle$ state is a linear combination of $|m\rangle \otimes |s_z\rangle$ states, but two of the states are just equal to a single $|m\rangle \otimes |s_z\rangle$ state. Which are they?

d) Starting from the one with the larger M, act on the equation $|J\ M\rangle = |m\rangle \otimes |s_z\rangle$ with $J_- = L_- + S_-$ (use J_- on the left and $L_- + S_-$ on the right) to determine how to write the other $|J\ M\rangle$ states for this J.

e) Can you find the remaining $|J\ M\rangle$ states using the fact that they must be orthogonal to the ones you found?