Homework sessions with TAs:

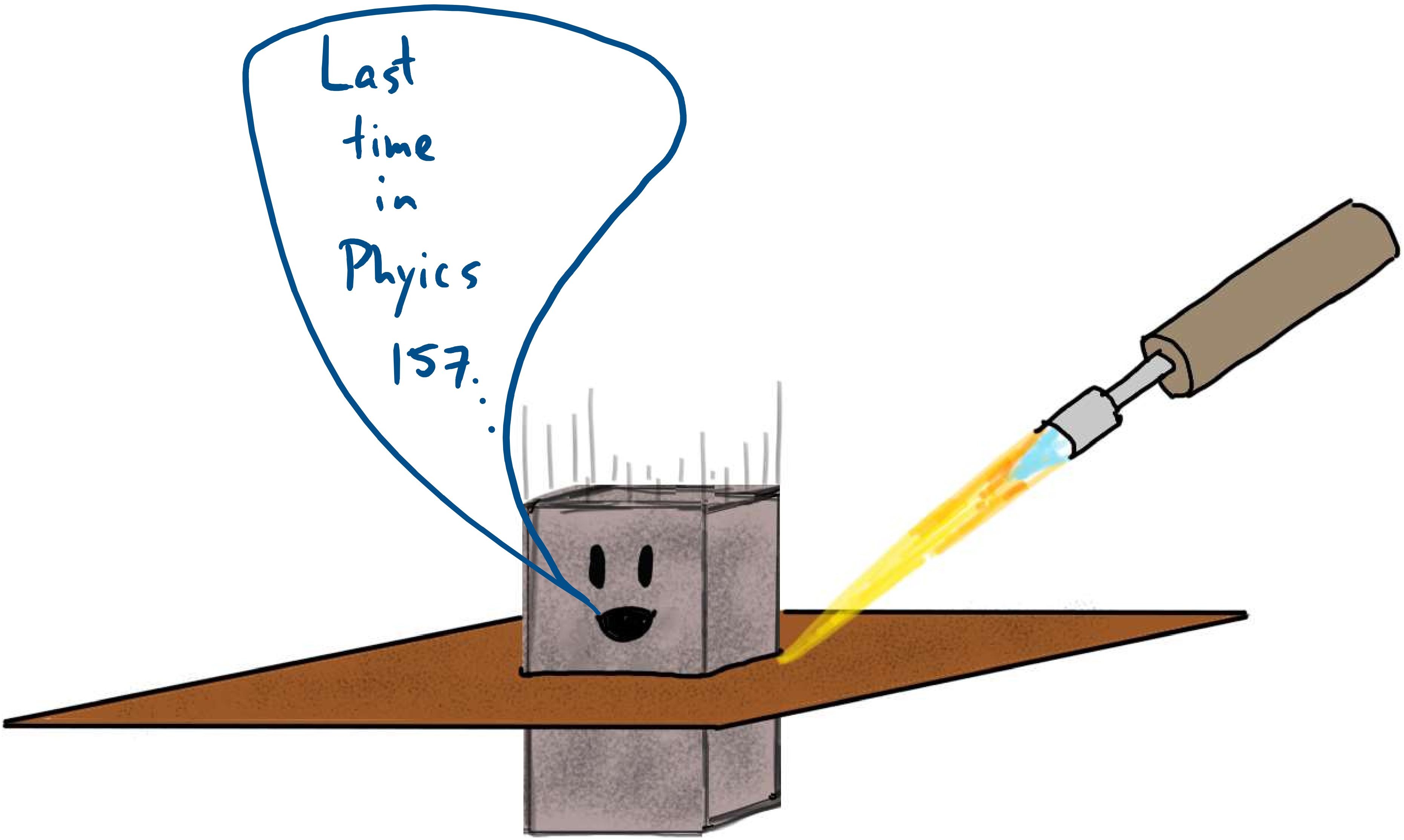
Monday 5-8 pm

Tuesday 5-8 pm

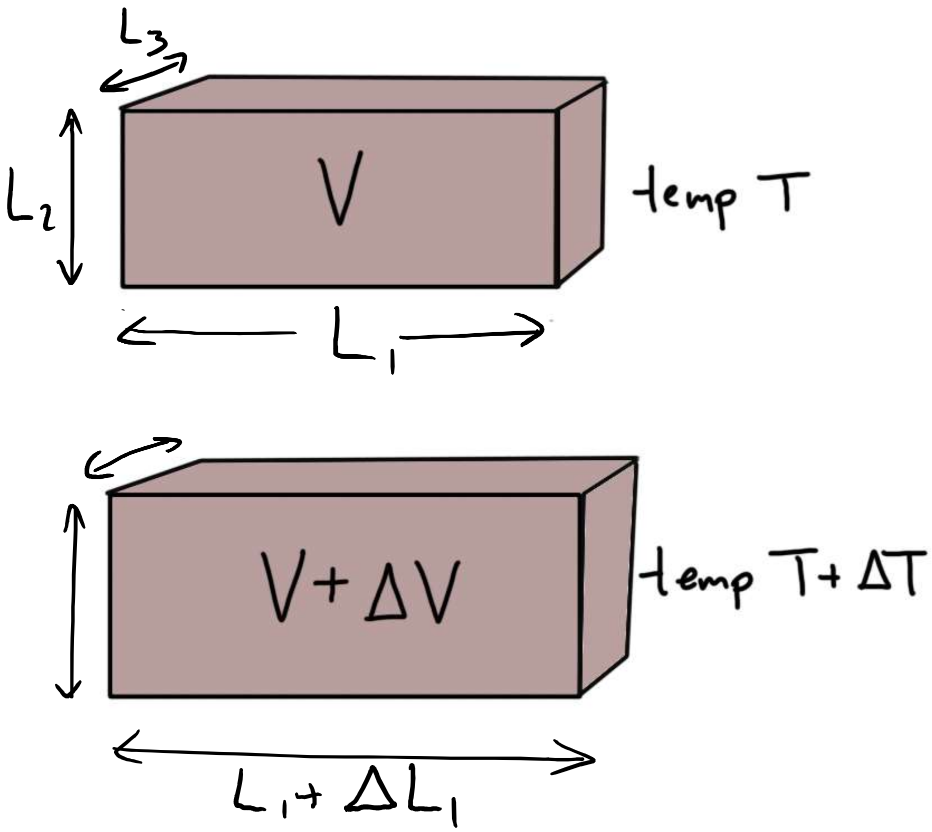
Office hours: After class, in Remo

4-5pm (Zoom)

8-9pm (Zoom)


Last time in Phyics 157...

Thermal expansion:



$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

applies to each dimension

$$\boxed{\Delta V = \beta V_0 \Delta T}$$

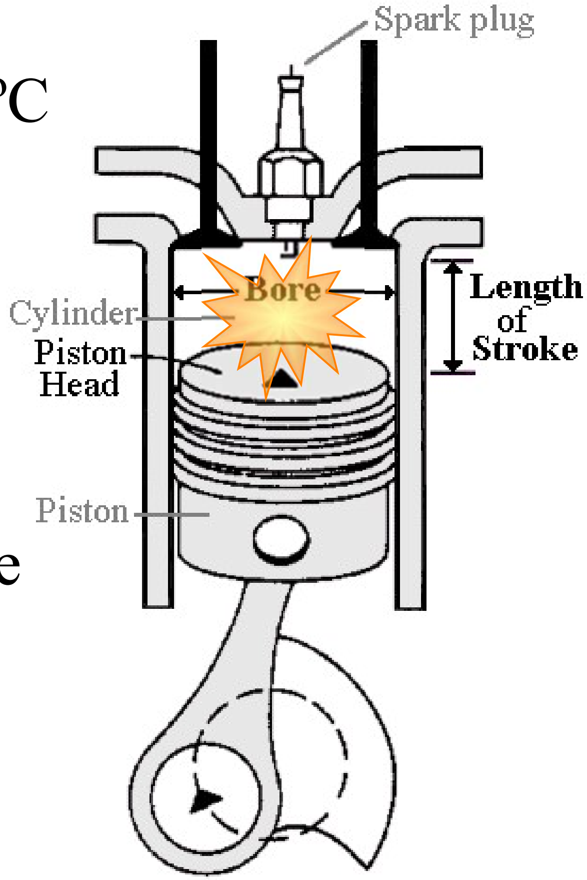
$\beta = 3\alpha$ for solids

also applies to liquids

Clicker: In some car engines, the piston is aluminum ($\alpha = 2.4 \times 10^{-5}$), while the cylinder is cast iron ($\alpha = 1.2 \times 10^{-5}$). If the engine needs to operator between 0ºC and 120ºC, which of these is **not** a good design:

A) The piston barely fits in the cylinder at 120ºC

B) The piston barely fits in the cylinder at 0ºC



EXTRA: what do we need to worry about if the engine gets too hot? Too cold?

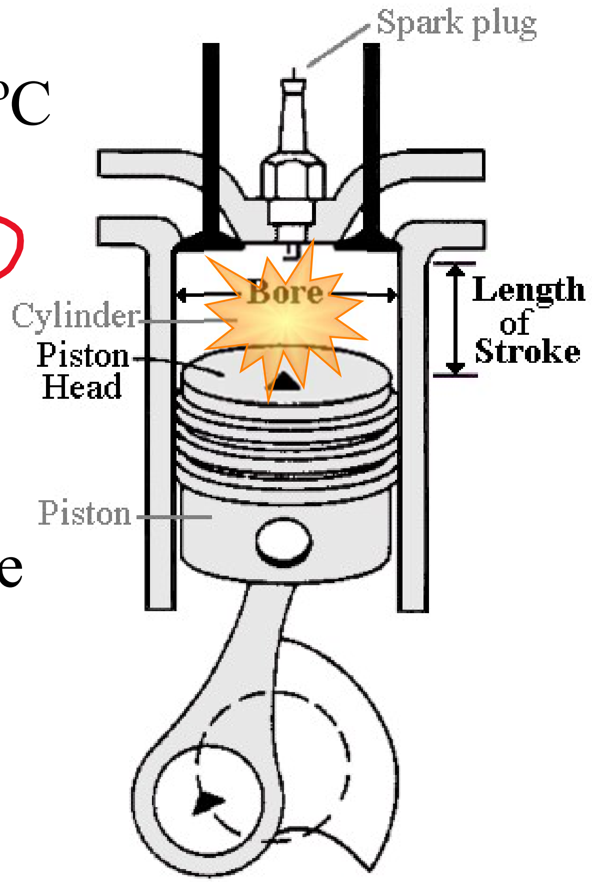
$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

Clicker: In some car engines, the piston is aluminum ($\alpha = 2.4 \times 10^{-5}$), while the cylinder is cast iron ($\alpha = 1.2 \times 10^{-5}$). If the engine needs to operator between 0ºC and 120ºC, which of these is not a good design:

A) The piston barely fits in the cylinder at 120ºC

[✓ B) The piston barely fits in the cylinder at 0ºC]

piston expands more than cylinder as engine heats up. Wouldn't be able to move at higher temperatures



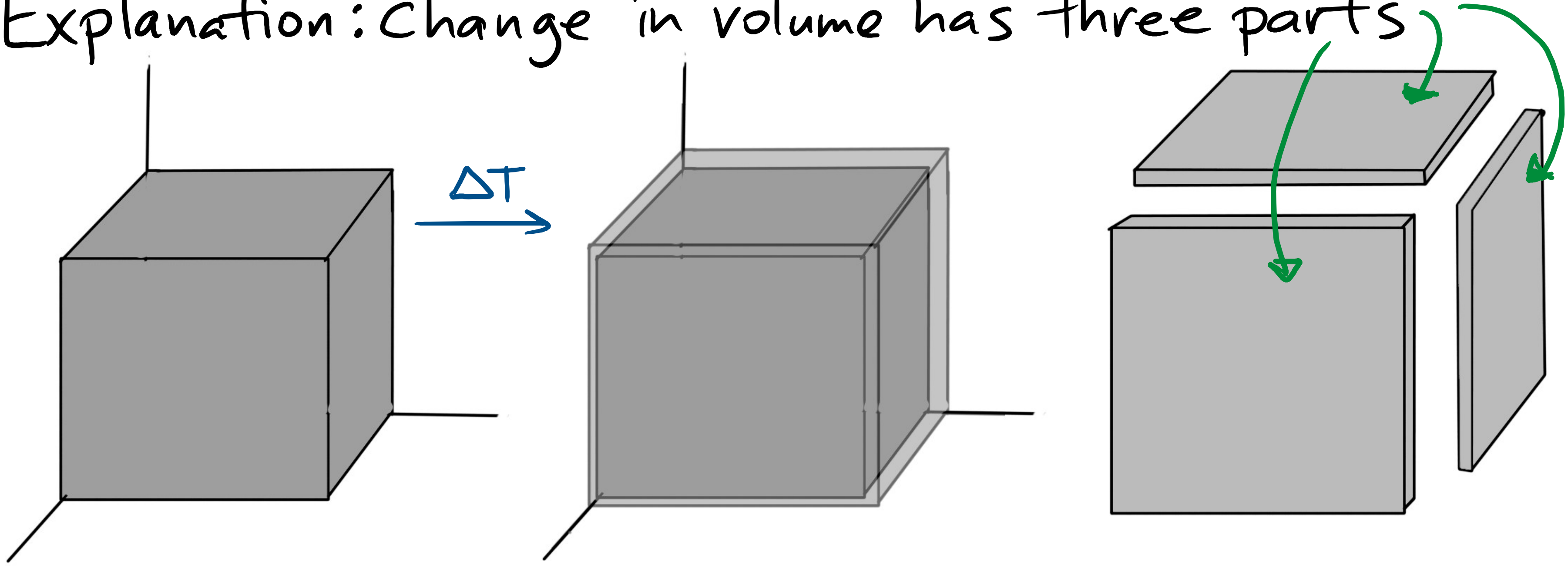
EXTRA: what do we need to worry about if the engine gets too hot? Too cold?

$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

Why is $\beta = 3\alpha$?

This means: for small $\frac{\Delta L}{L}$, percentage change in volume is 3 times percentage change in length

Explanation: Change in volume has three parts



If length increases by X%, each part is X% of original volume.

some small number

Mathematical derivation:

original volume: L^3

new volume $(1.001 \times L)^3 \approx 1.003\ L^3$

so 0.3% bigger

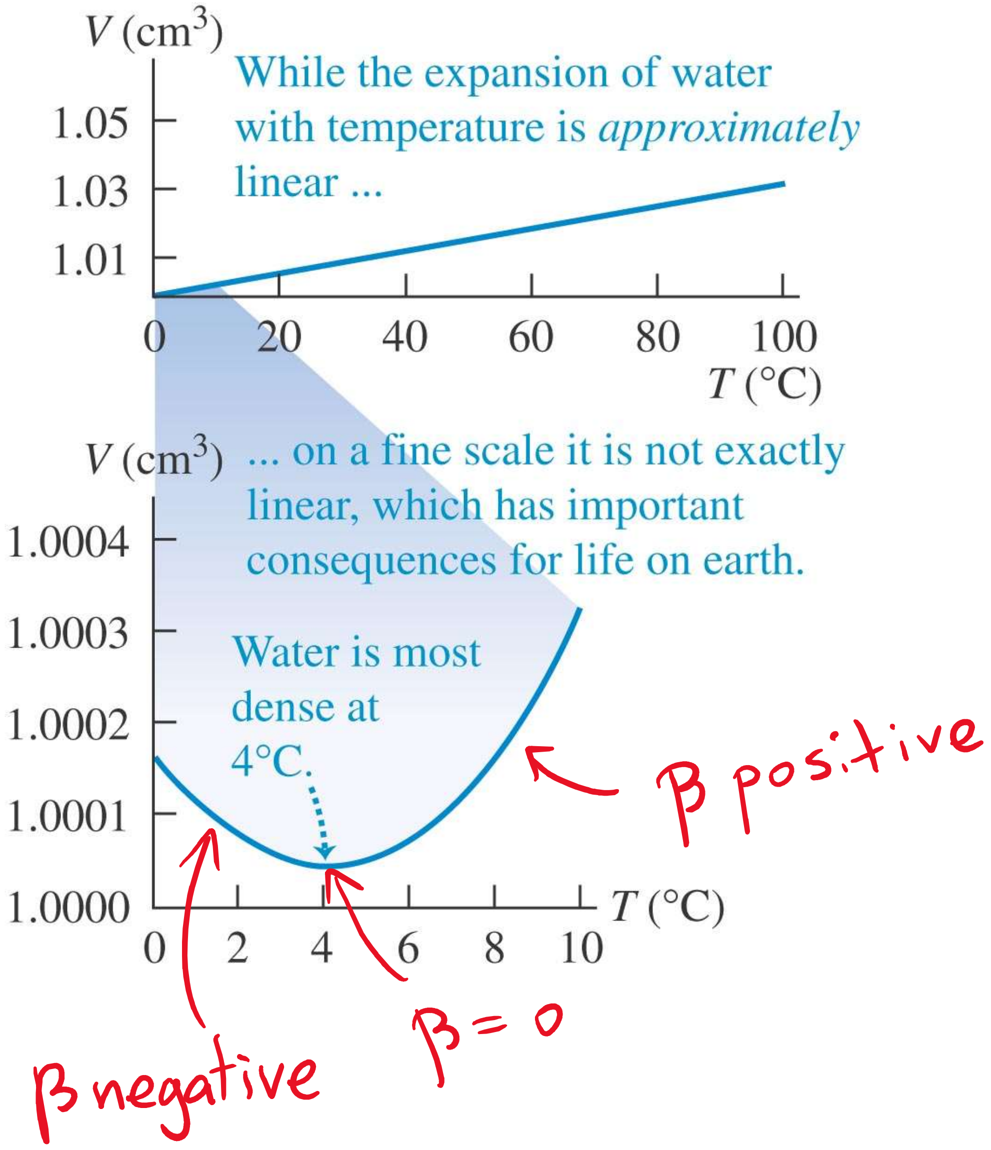
generally: $$(L+\Delta L)^3 = \underset{V}{L^3} + \underbrace{3L^2\Delta L + 3L(\Delta L)^2 + (\Delta L)^3}_{\Delta V}$$

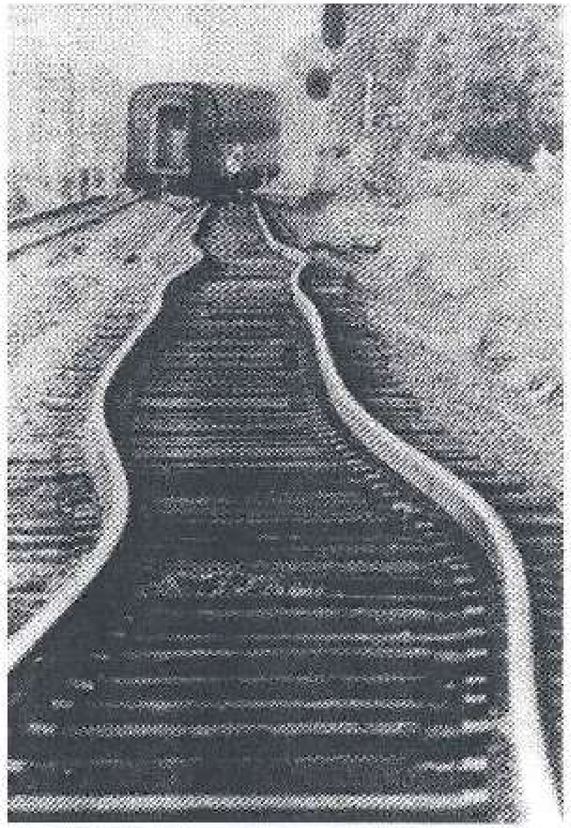
$$\frac{\Delta V}{V} = 3\cdot\frac{\Delta L}{L} + 3\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta L}{L}\right)^3$$

this means $\beta = 3\alpha$

these are negligible compared to the first term if $\frac{\Delta L}{L}$ is small

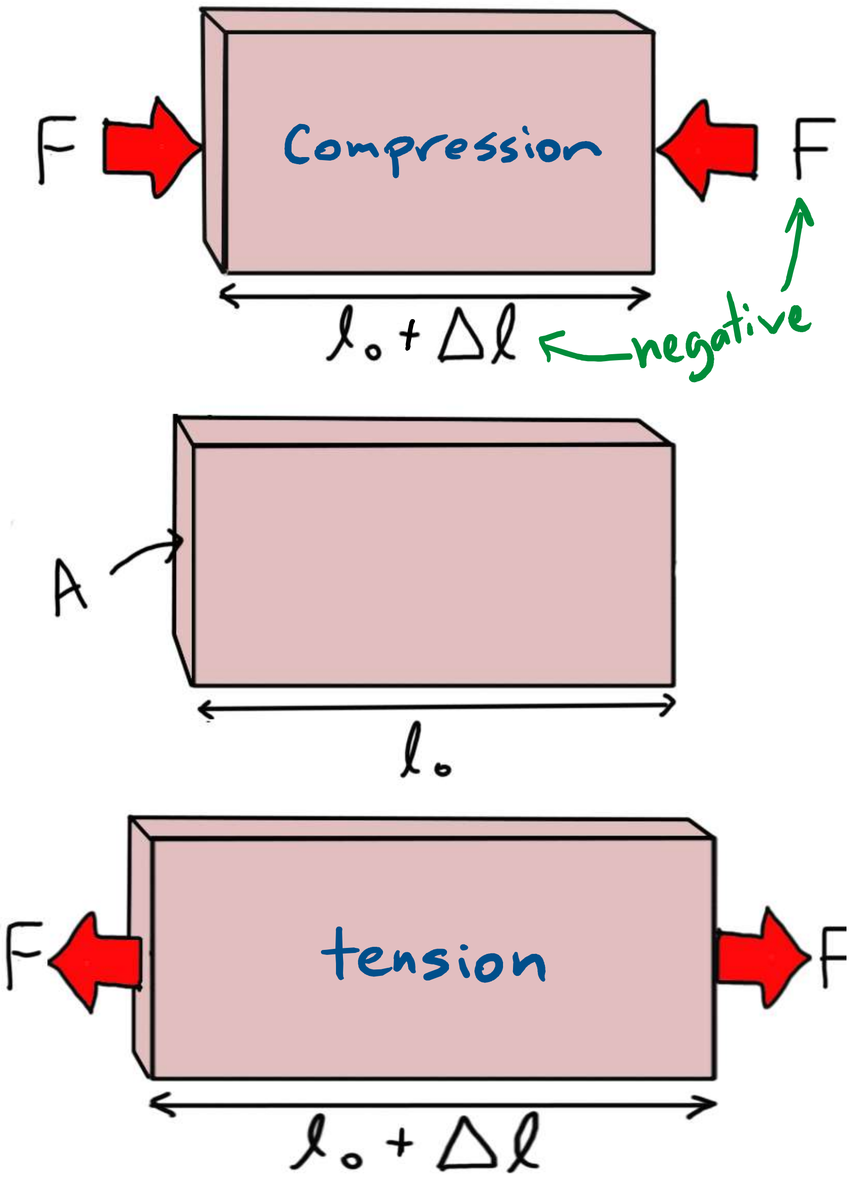
- Water, a special example





Thermal expansion can lead to large forces!

STRESS & STRAIN



$$\frac{F}{A} = Y \frac{\Delta l}{l_0}$$

strain

stress (units of pressure)

Young's modulus

"

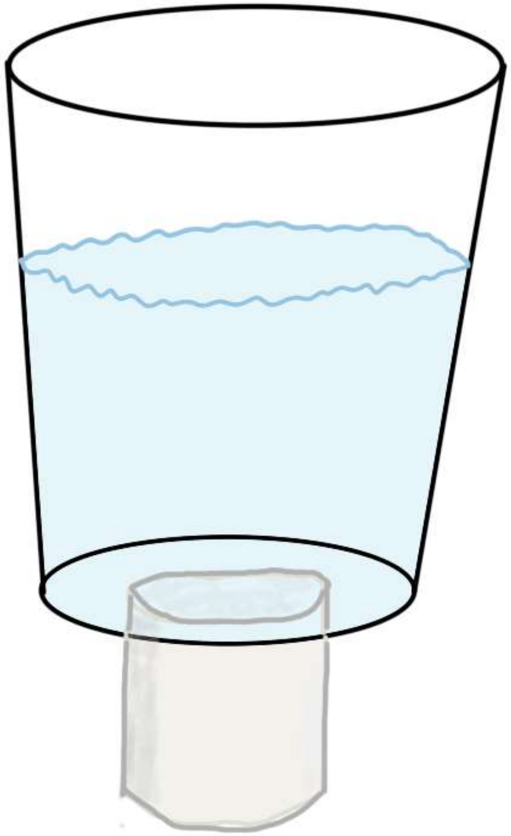
a basic property of a material (resistance to squishing)

like $F = k \Delta x$ for spring

Young's modulus of a marshmallow

$$\frac{F}{A} = Y \frac{\Delta L}{L_0}$$

$$\frac{\Delta L}{L_0} \approx 0.1$$



$$F \approx 1\,N$$

$$A \approx 5\ cm^2 = 5 \times (10^{-2} m)^2$$

gives $Y \approx 2 \times 10^4\ N/m^2$

Clicker: Suppose you repeated the measurement of Y for a mini-marshmallow. In this case, we would expect a value of Y that is

A. Significantly higher
B. Significantly lower
C. About the same

$$\boxed{\frac{F}{A} = Y \frac{\Delta L}{L_0}}$$

Clicker: Suppose you repeated the measurement of Y for a mini-marshmallow. In this case, we would expect a value of Y that is

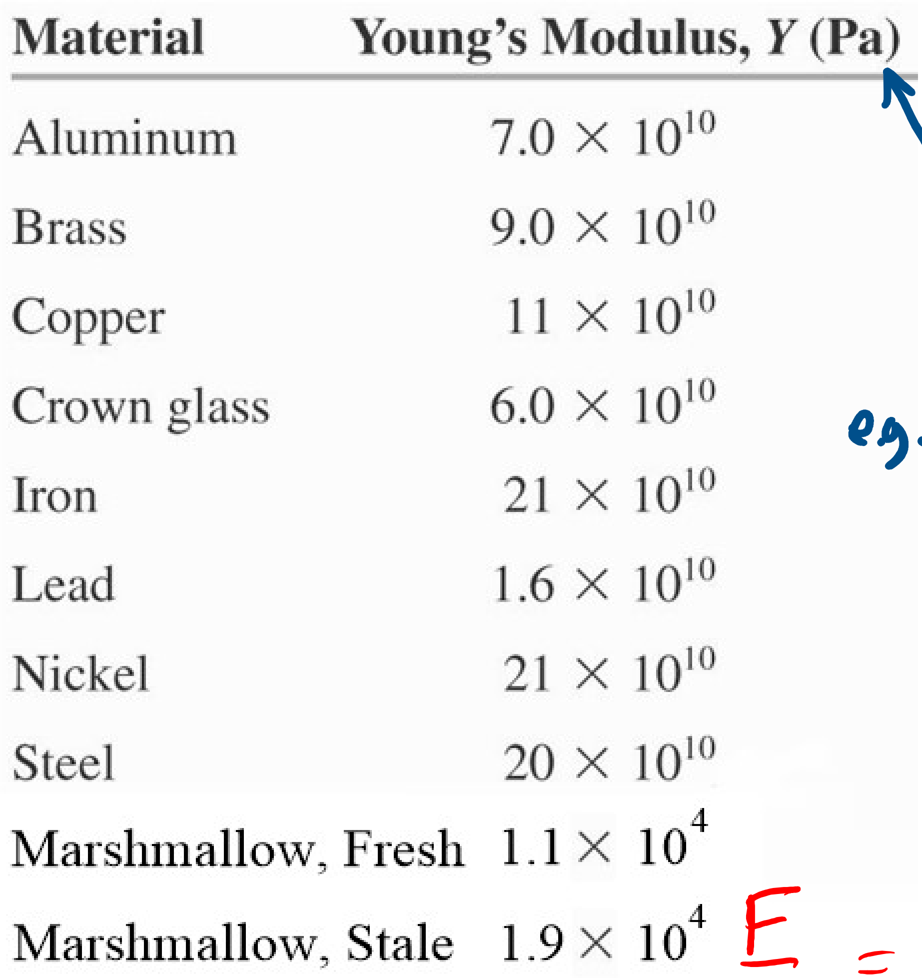
A. Significantly higher

B. Significantly lower

C. **About the same: Y is just a property of the material and doesn't depend on size and shape**

$$\boxed{\frac{F}{A} = Y \frac{\Delta L}{L_0}}$$

Young Modulus of Various Materials



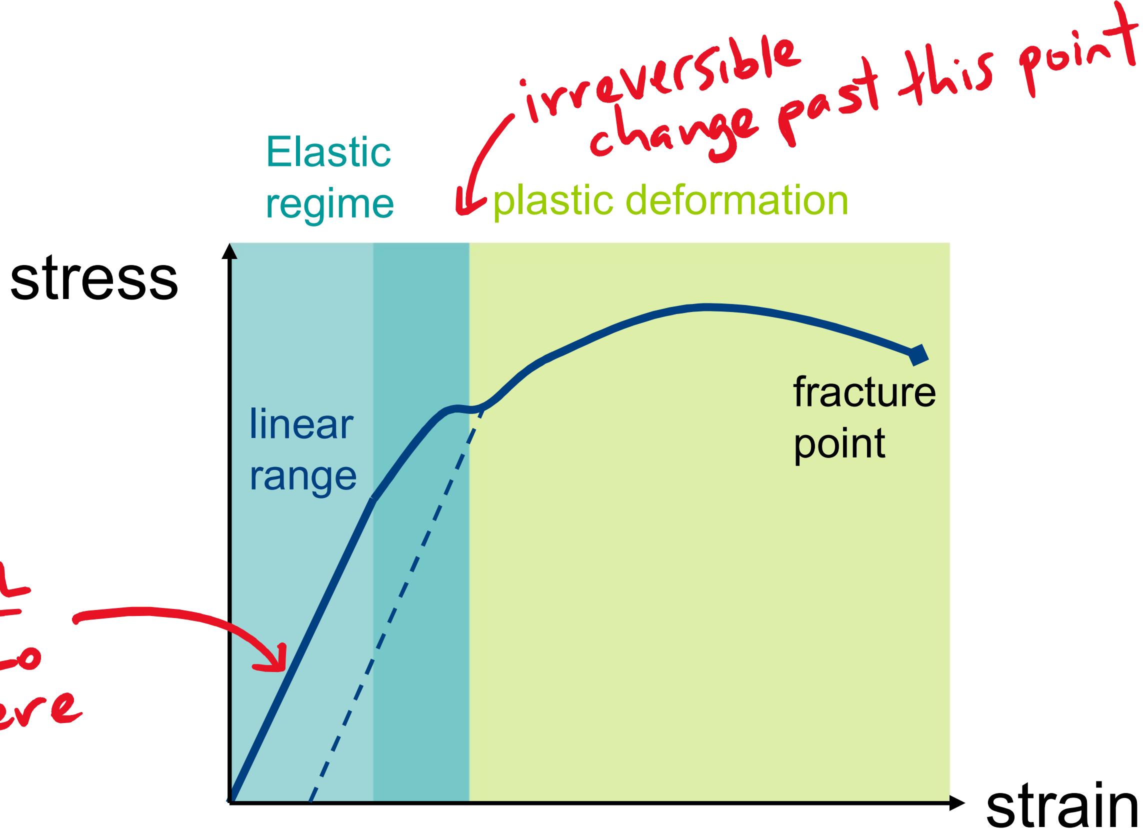
Material	Young's Modulus, *Y* (Pa)
Aluminum	7.0×10^{10}
Brass	9.0×10^{10}
Copper	11×10^{10}
Crown glass	6.0×10^{10}
Iron	21×10^{10}
Lead	1.6×10^{10}
Nickel	21×10^{10}
Steel	20×10^{10}
Marshmallow, Fresh	1.1×10^{4}
Marshmallow, Stale	1.9×10^{4}

units of pressure:

eg. Pressure of 0.01% of Y on ends will give 0.01% compression

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

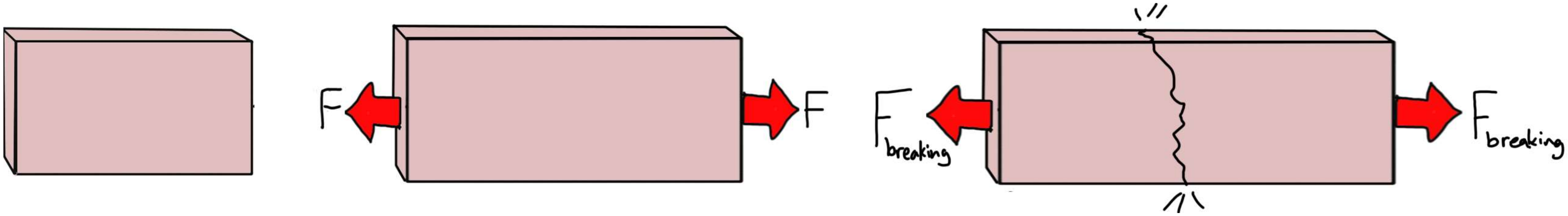
Elastic vs. Plastic



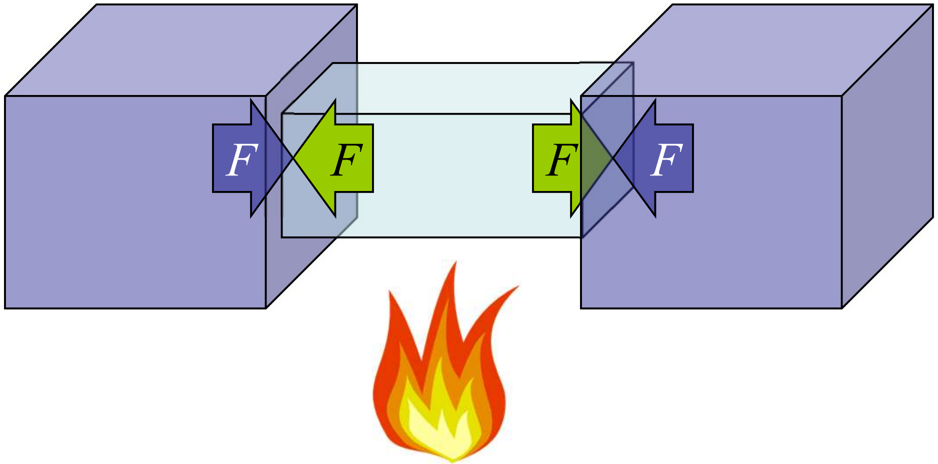
irreversible change past this point

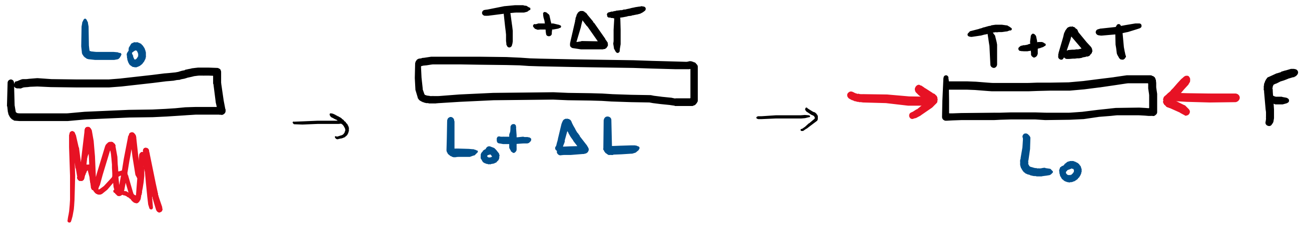
$$\frac{F}{A} = Y \frac{\Delta L}{L_o}$$

valid here



THERMAL STRESS : forces on a material due to surrounding materials preventing thermal expansion/contraction





A steel rod of length L_0 is heated by temperature ΔT.

How much stress (force per unit area) is required to compress the rod back to its original length?

Write an answer for the magnitude of F/A in terms of ΔT and the parameters Y, α, L_0 for the rod.

Click A if you have an answer, B if you are stuck.

thermal expansion:

$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

stress vs strain:

$$\boxed{\frac{F}{A} = Y \frac{\Delta L}{L_0}}$$

A steel rod of length L_0 is heated by temperature ΔT and expands. How much stress (force per unit area) is required to compress the rod back to its original length?

Write an answer for the magnitude of F/A in terms of Y, α, $L_{0,}$ and ΔT.

A) $Y \alpha L_0 \Delta T$

B) $Y \alpha \Delta T$

C) $Y L_0 \Delta T$

D) $\alpha L_0 \Delta T$

E) $Y \alpha L_0$

We have $\Delta L_{Th} = \alpha L_0 \Delta T$

We need the change in length due to stress to be the negative of this:

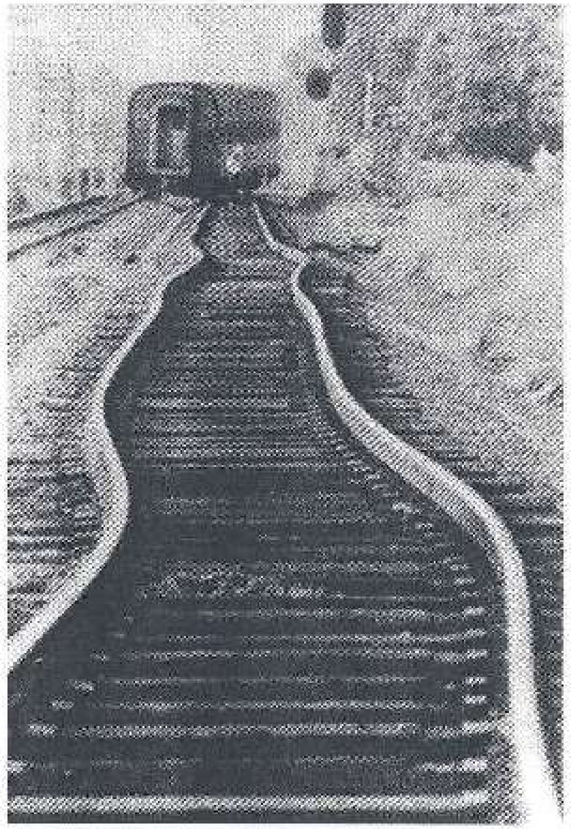
$\Delta L_{St} = -\Delta L_{Th}$

The stress is then:

$$F/A = Y \Delta L_{St} / L_0$$

$$= - Y \Delta L_{Th} / L_0$$

$$= - Y\alpha \Delta T$$



Clicker: 10m long steel train rails are laid end to end on a winter day (0 ºC). If the engineer forgot to leave gaps for thermal expansion, roughly how much force is generated at the ends of each rail due to thermal stress when the temperature reaches 30 ºC?

Cross sectional area of rail: 0.01m²

$Y_{steel} = 20 \times 10^{10}\ Pa$

$\alpha_{steel} = 1.2 \times 10^{-5}\ K^{-1}$

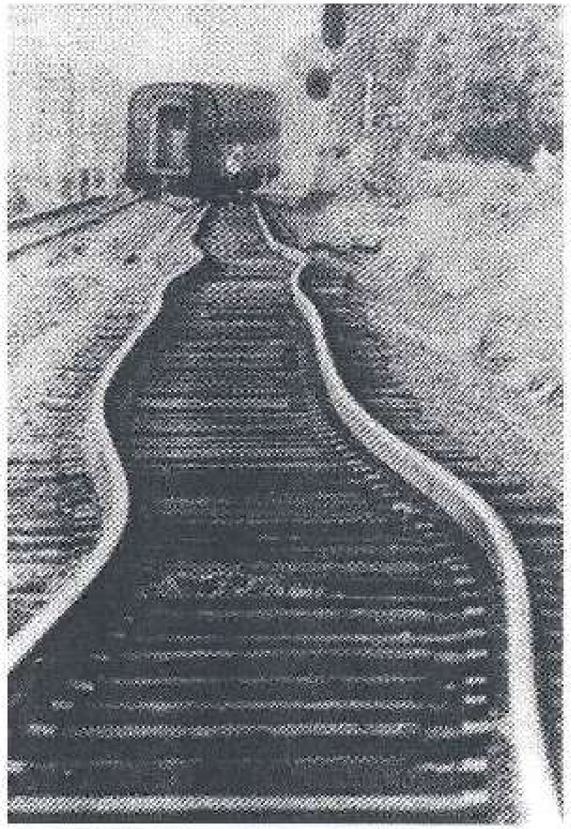
A) 700 N B) 7,000 N C) 70,000 N

D) 700,000 N E) 7,000,000 N

previous result:

$$\frac{F}{A} = Y\alpha\,\Delta T$$

EXTRA: How much gap should have been left?



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EXTRA: How much gap should have been left?


fully loaded Boeing 737
AIR CANADA
rail