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original volume: L^3

new volume $(1.001 \times L)^3 \approx 1.003\ L^3$

so 0.3% bigger

generally: $(L+\Delta L)^3 = L^3 + \underbrace{3L^2\Delta L + 3L(\Delta L)^2 + (\Delta L)^3}_{\Delta V}$

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$$\frac{\Delta V}{V} = 3\cdot\frac{\Delta L}{L} + 3\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta L}{L}\right)^3$$

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