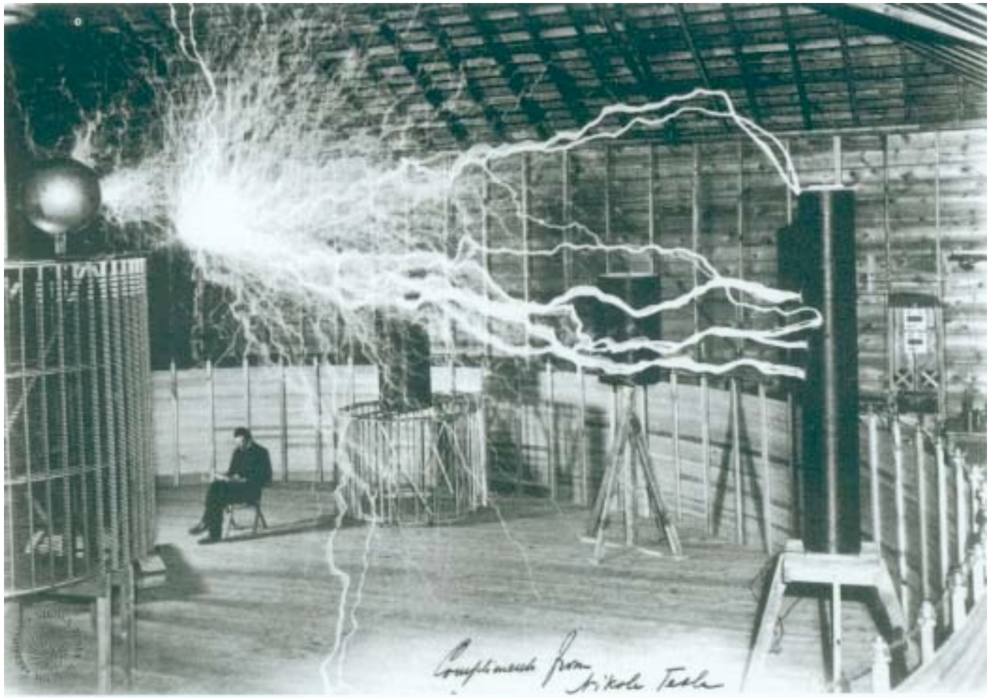
Circuit Analysis



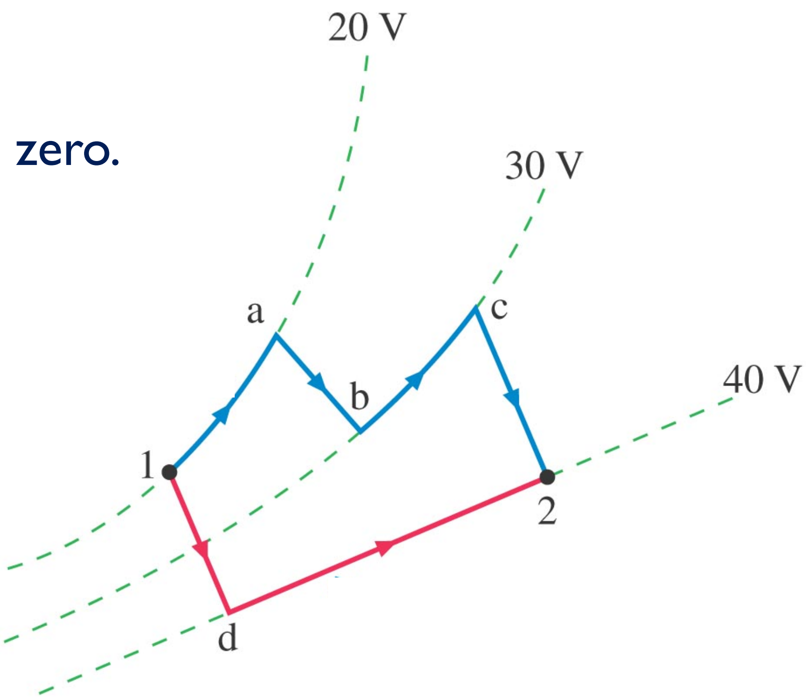
Kirchhoff's Loop law

A foundation of circuit analysis.

The loop law comes from path independence:

$$\Delta U = -\oint \vec{F} \cdot d\vec{s} = -q \oint \vec{E} \cdot d\vec{s} = 0$$

The change in energy around a closed path is zero.



We then know that $U = qV$ gives

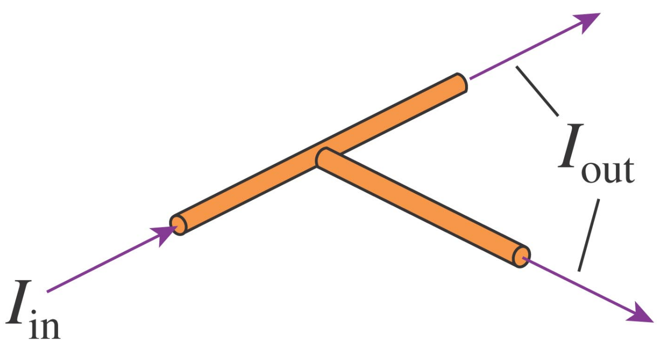
$$\Delta V_{\text{loop}} = 0$$

Which is the loop law.

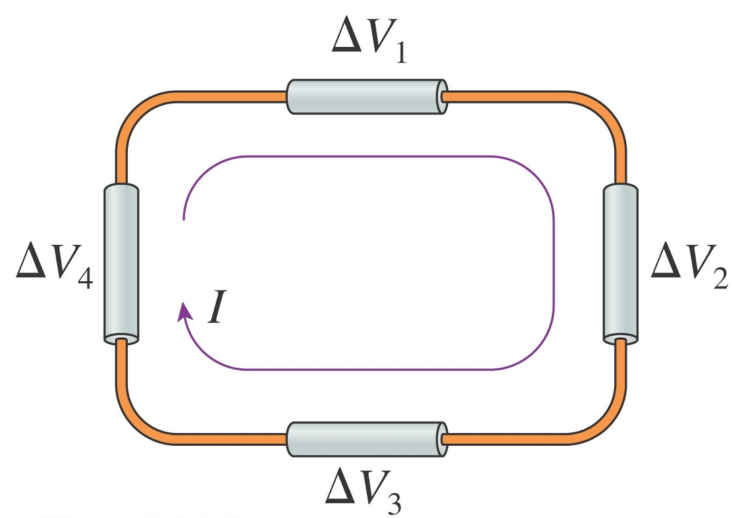
Kirchhoff's Laws

Kirchhoff's Laws are summarized as:

$$\sum I_{in} = \sum I_{out} \quad \text{at a junction}$$



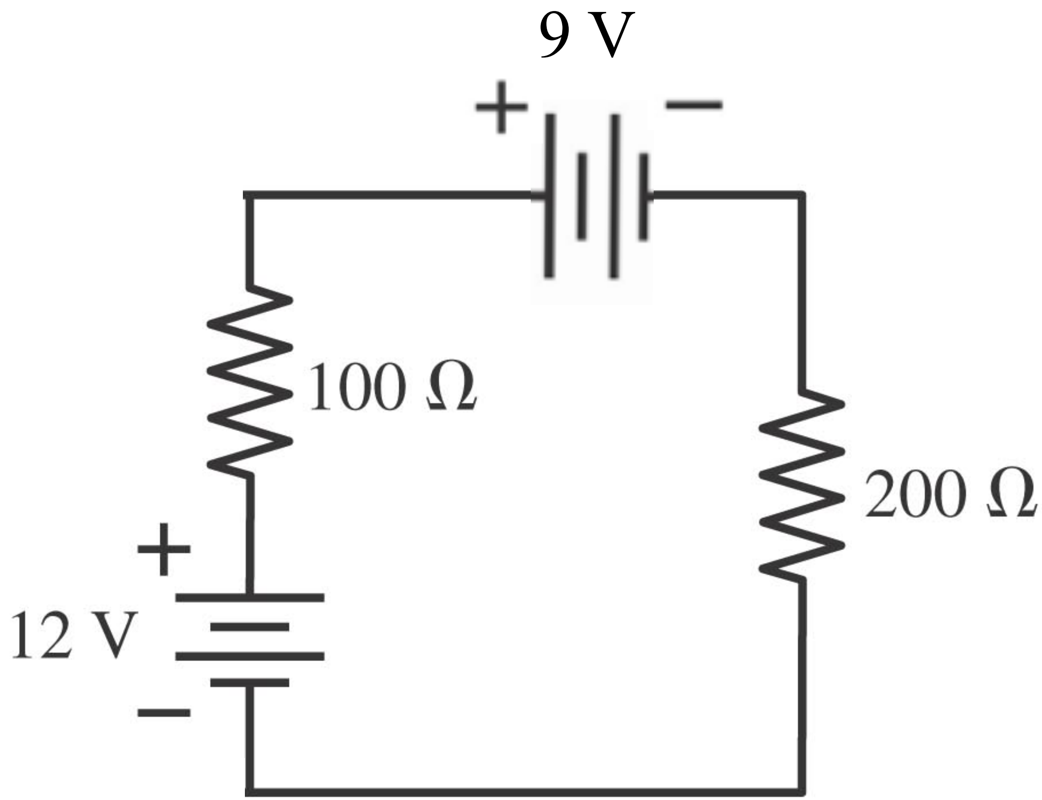
$$\sum_i \Delta V_i = 0 \quad \text{around closed loop}$$



$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$

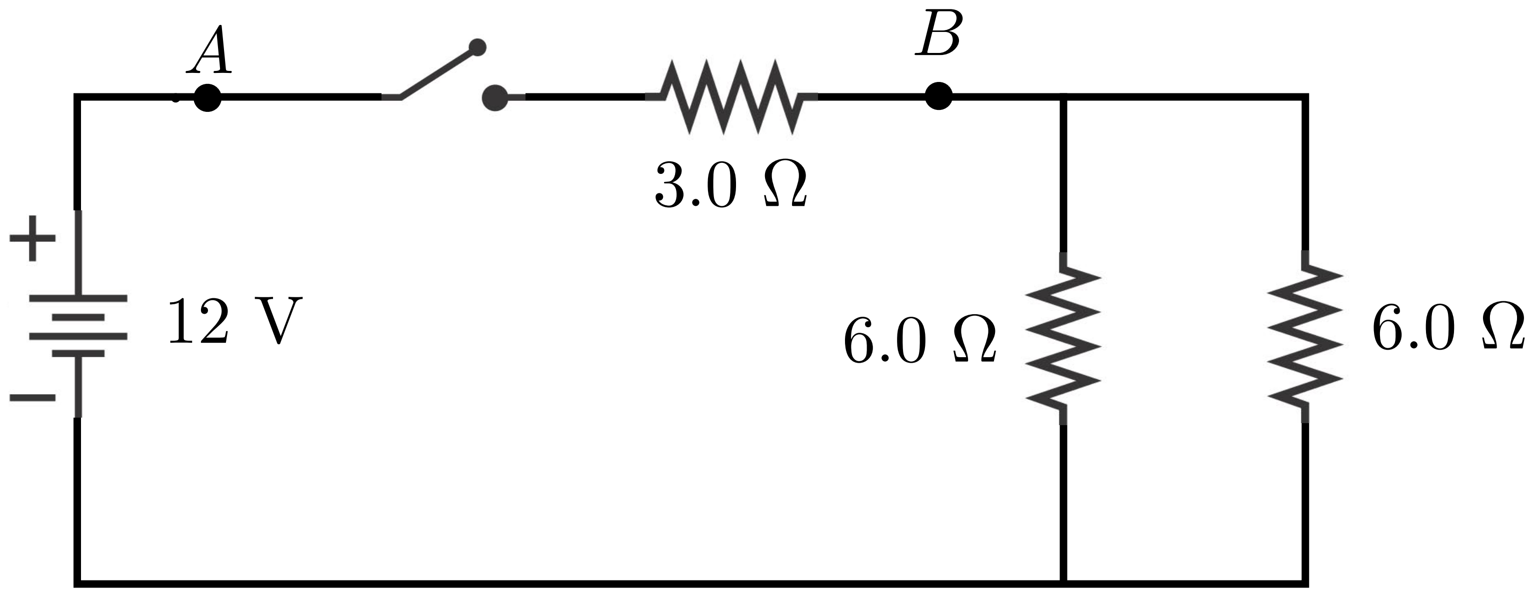
The Loop Law in Action

Find the current running through the circuit.



Clicker Question

The switch is open. What is the potential difference between point *A* and *B?*



a) 0 V
b) 3 V
c) 6 V
d) 9 V
e) 12 V

Clicker Question

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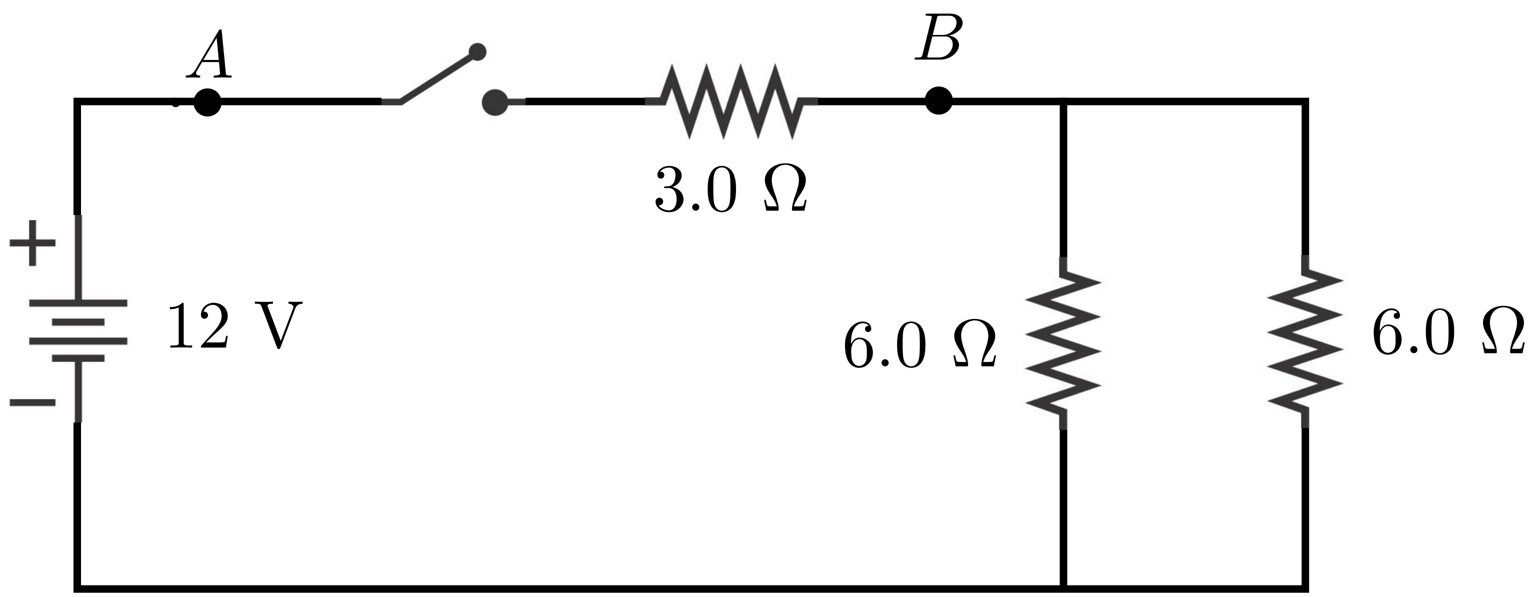

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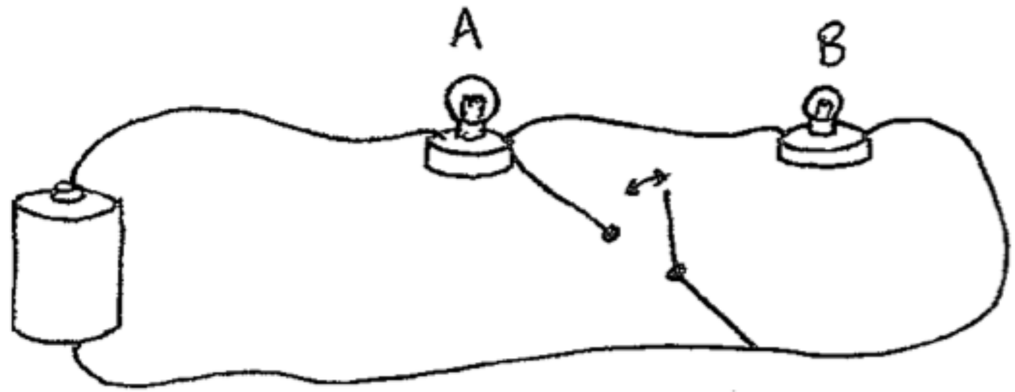
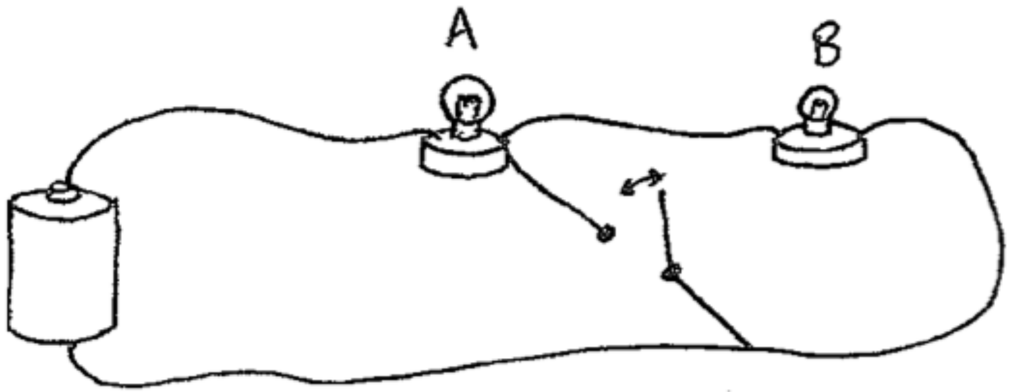
Worksheet Questions 5, 6, and 7



The circuit starts with the switch *closed*. What happens to bulb B when the switch is *opened*?

a) It gets brighter
b) It stays the same brightness
c) It gets dimmer
d) It goes out completely



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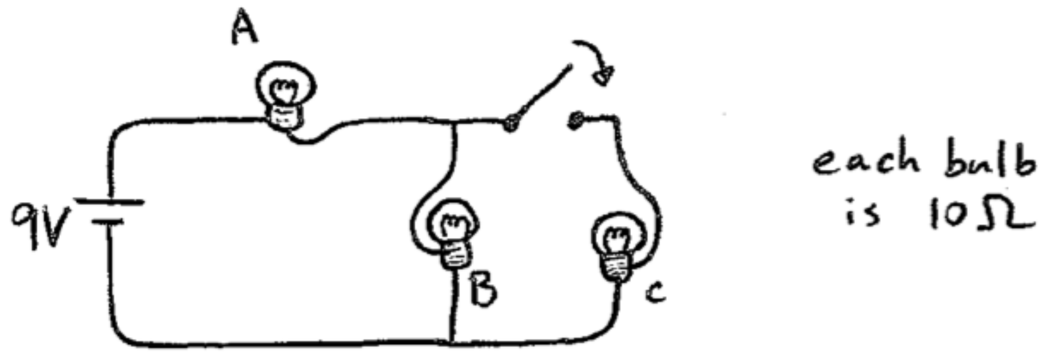
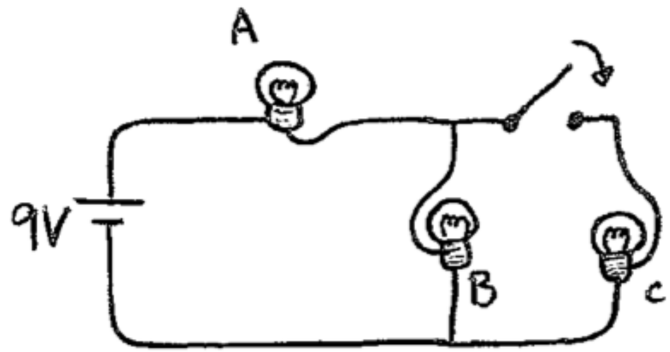


The circuit starts with the switch *open*. What happens to bulb A when the switch is *closed*?

a) It gets brighter
b) It stays the same brightness
c) It gets dimmer
d) It goes out completely

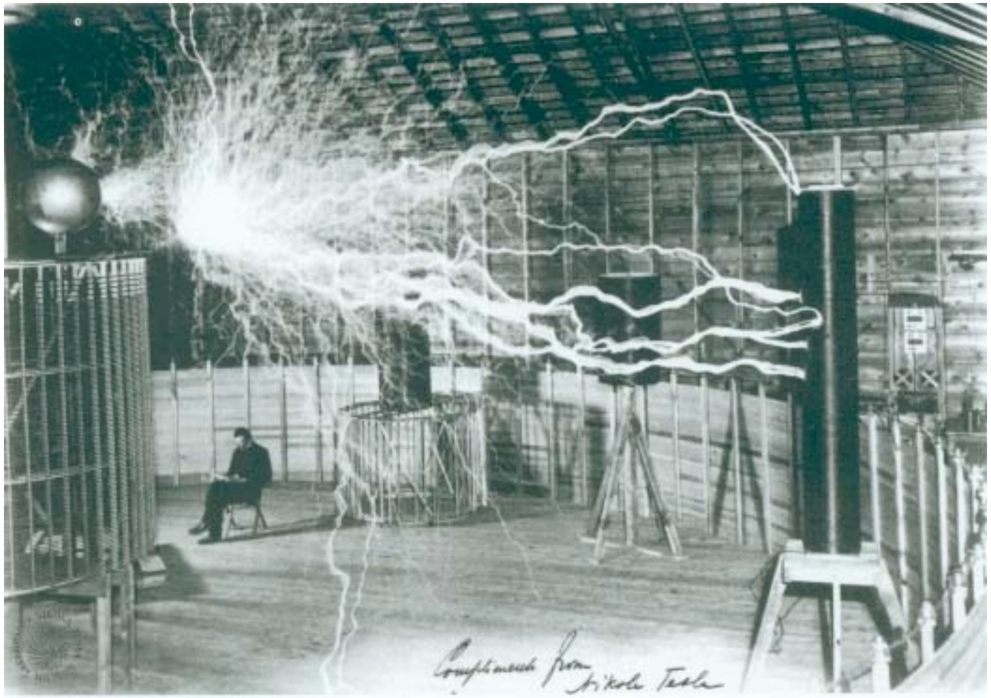


each bulb
is 10Ω

The circuit starts with the switch *open*. What happens to bulb A when the switch is *closed*?

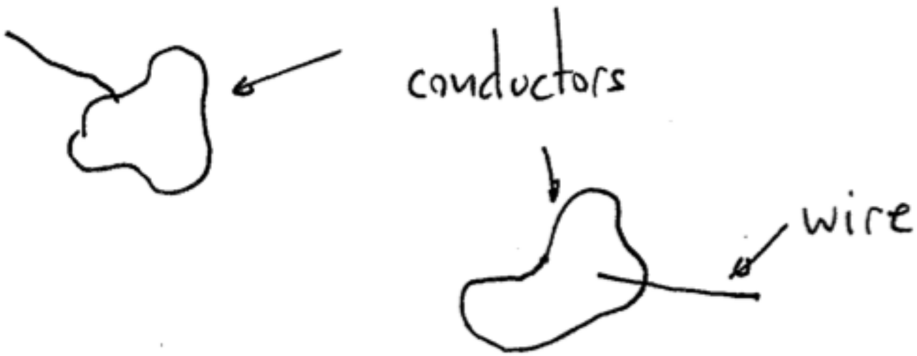
a) It gets brighter
b) It stays the same brightness
c) It gets dimmer
d) It goes out completely

Capacitance



What is a Capacitor?

Two conductors separated by some distance.

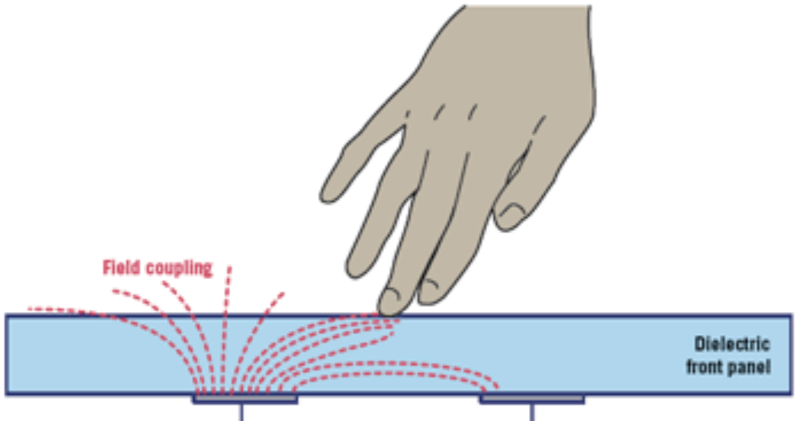


Most common configuration is the parallel plate capacitor.

The capacitor is a **physical break in the circuit**!

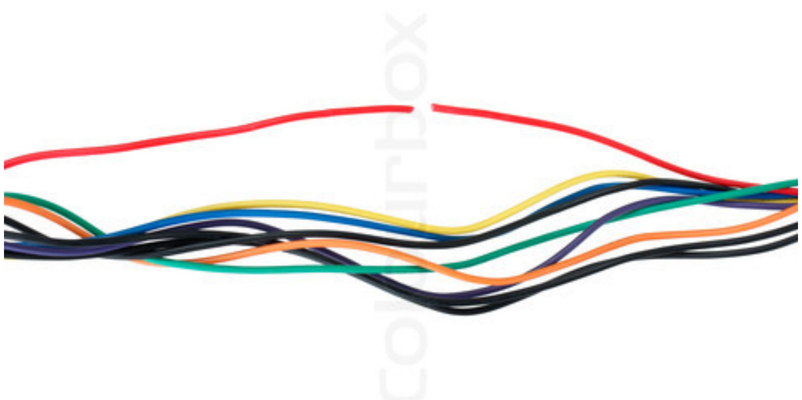
Examples of Capacitors

Examples of Capacitors

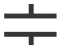


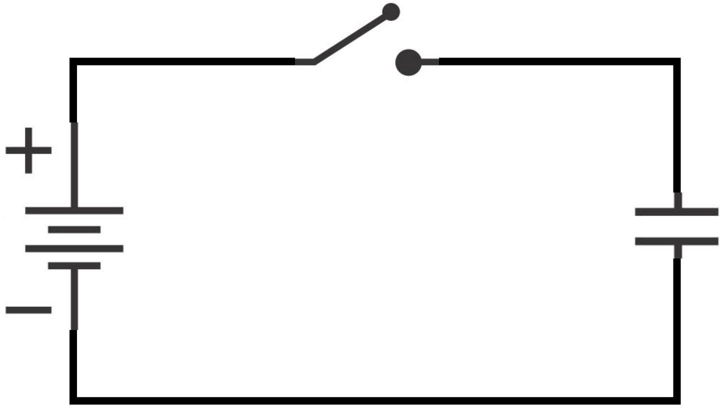
Conductive finger and conductive layer in a ***touch screen*** form a capacitor.

A **break** in a wire or a bad connection forms a capacitor.



Clicker Question

As described, the capacitor is essentially a break in the circuit (it's even drawn that way: ).



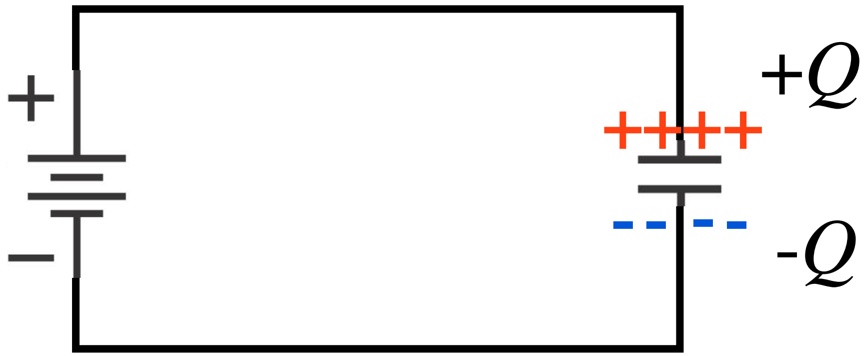
Does current flow when the switch is closed?

a) yes
b) sort of
c) no

AC/DC Circuit PhET #1 and #2

Clicker Question

The current **flows**, then **stops**.



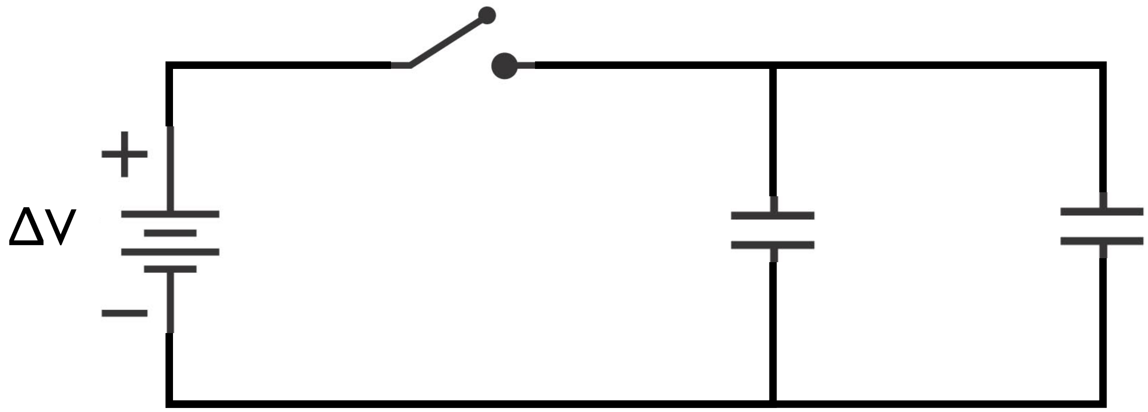
Current stops because the **battery can't do any more work**.

This is called "charging" the capacitor with **charge Q**, even though the capacitor's **total charge is zero**.

AC/DC Circuit PhET #1 and #2

Observation

Watch closely. What happens now when we close the switch?



AC/DC Circuit PhET #1 and #2

Observation

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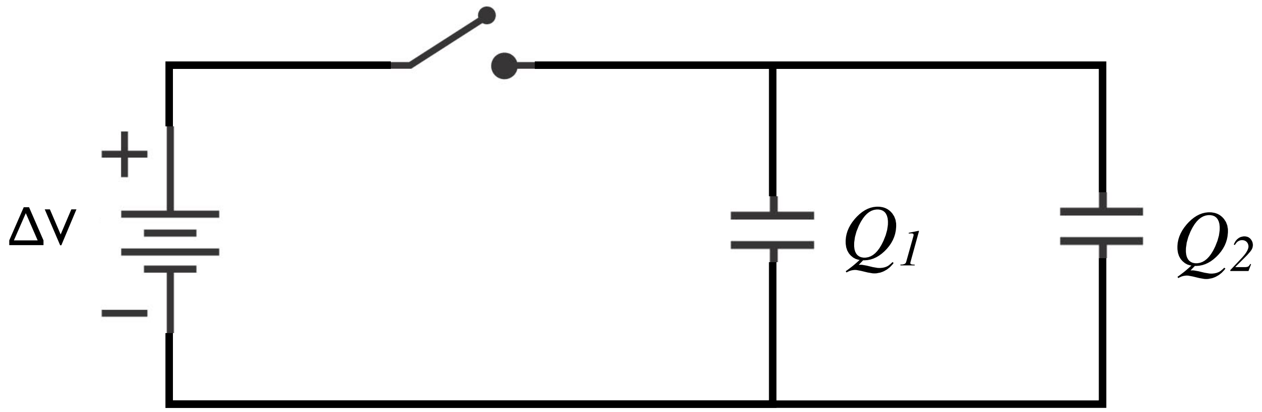


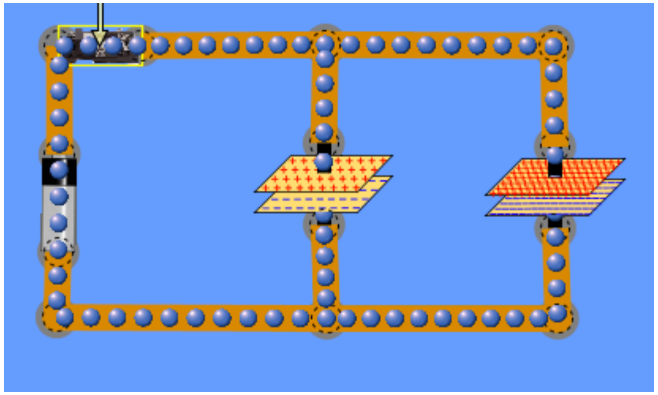
1. The capacitors have **different charges Q_1 and Q_2**.
2. The capacitors have the **same voltage ΔV** across them.
3. The ratio **Q**/**ΔV** is different for each capacitor.

AC/DC Circuit PhET #1 and #2

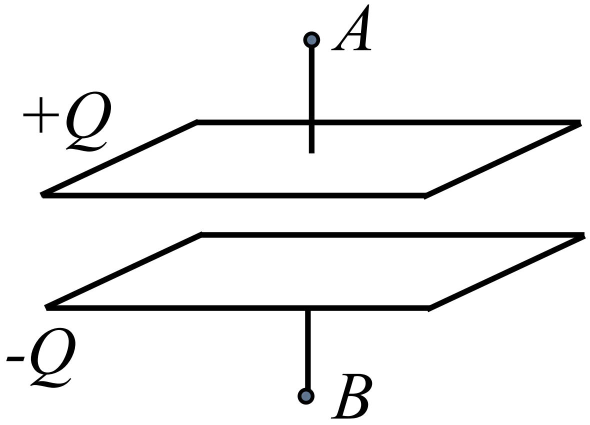
Capacitance

The ability for a capacitor to hold charge for a given voltage.

$$C = \frac{Q}{V}$$



where *V* is the voltage across the capacitor and +*Q* and -*Q* are the charges on the plates.



$$V = |\Delta V_{AB}|$$

Uses of Capacitors

Particle Accelerators

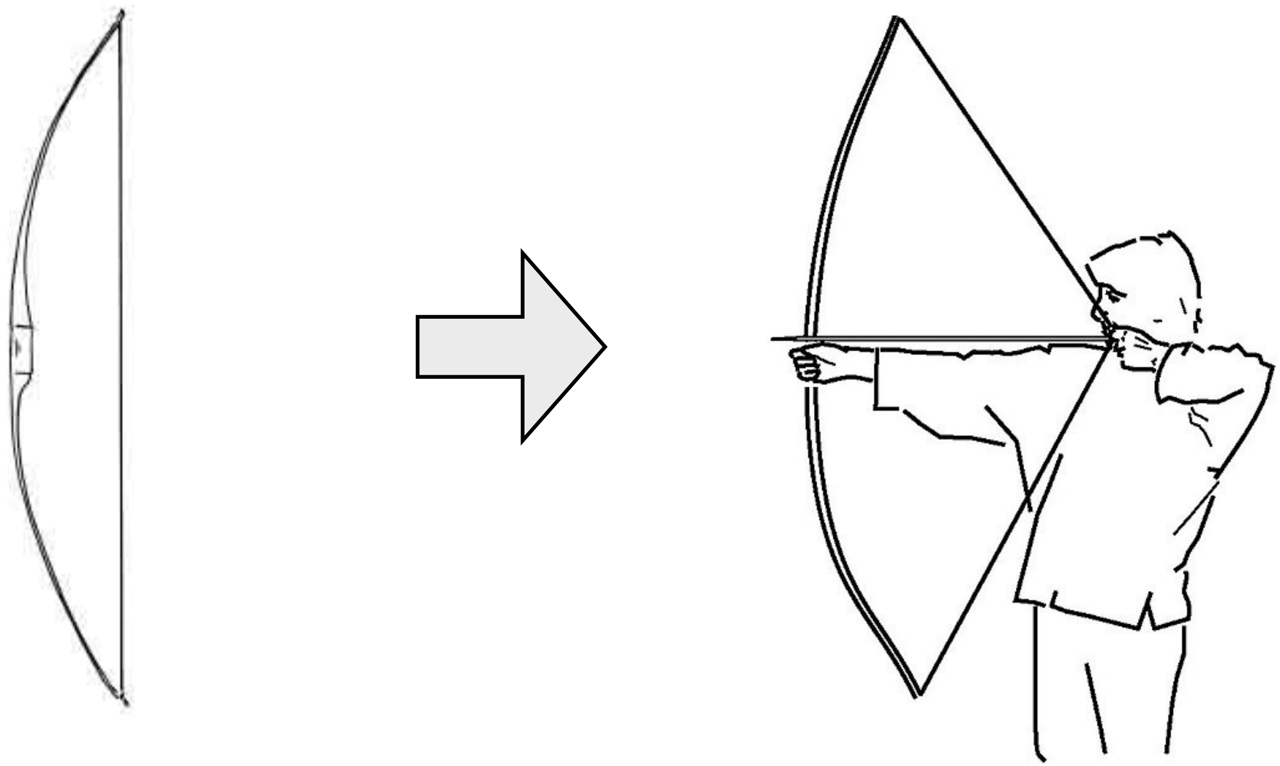
Lightning

RAM

Defibrillators

Storing Potential Energy

Pulling on a bow string increases potential energy



Shoots arrows faster than we can throw them.

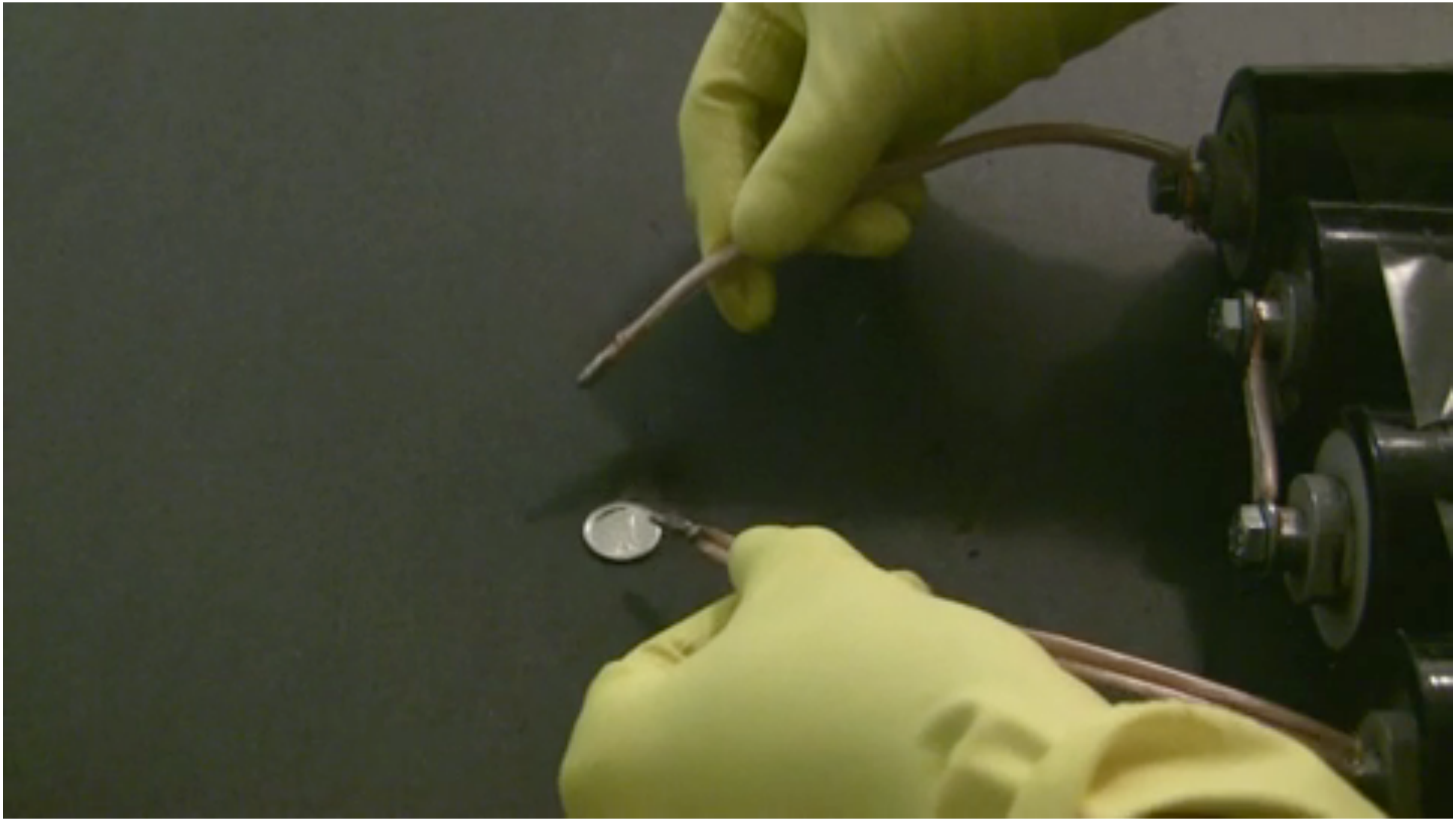
Storing Potential Energy

Lifting a clam increases potential energy

Birds drop clams to break them open.

Capacitors Release Energy Quickly

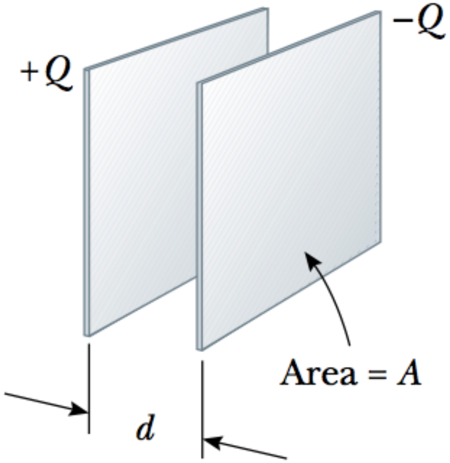
A 9V battery can't do this.



fun with ultacapacitors (video)

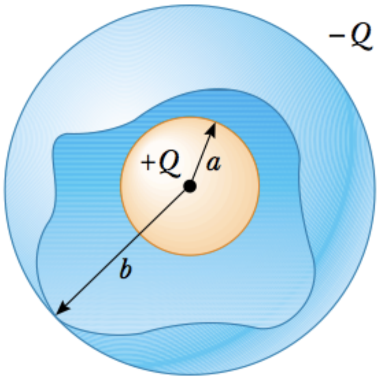
Capacitance is Geometric

Parallel Plate
(from assignment)



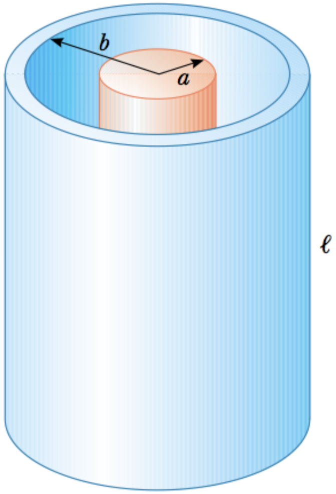
$$C = \frac{\epsilon_0 A}{d}$$

Spherical



$$C = 4\pi\epsilon_0 \left(\frac{ab}{b-a}\right)$$

Cylindrical



$$C = 2\pi\epsilon_0 \frac{\ell}{\ln\left(b/a\right)}$$

Clicker Question

A parallel-plate capacitor is ***charged*** by a 12V battery and then ***disconnected from it***. Pressing on the capacitor, i.e., reducing the gap between the plates, leads to

	C	ΔV	Q
A	+	+	+
B	+	+	-
C	+	=	+
D	+	-	=
E	=	=	=

+ increase
- decrease
= no change

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	C	ΔV	Q
A	+	+	+
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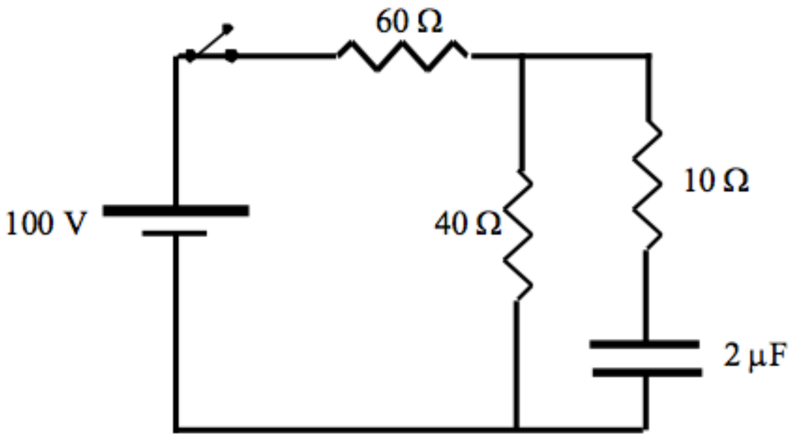
C	+	=	+
D	+	-	=
E	=	=	=

\+ increase
\- decrease
= no change

This is how a pressure plate sensor and some microphones work.

Clicker Question

We close the switch. What is the voltage across the capacitor once the circuit has run for a while?



a) 0 V
b) 11.8 V
c) 40 V
d) 60 V
e) Unable to determine.

Clicker Question

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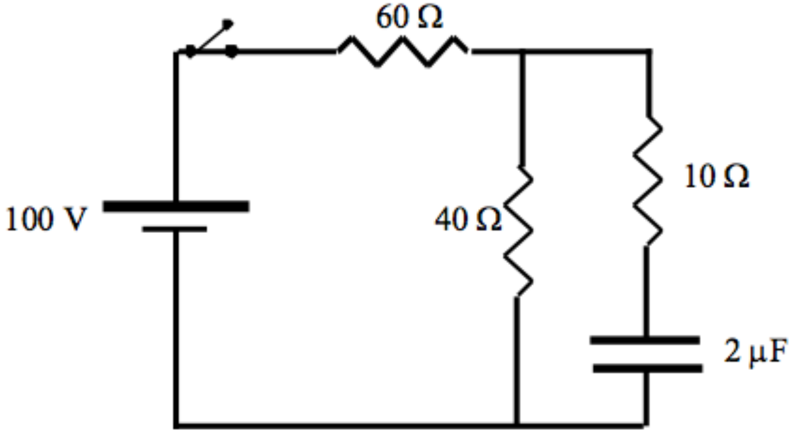

a) 0 V
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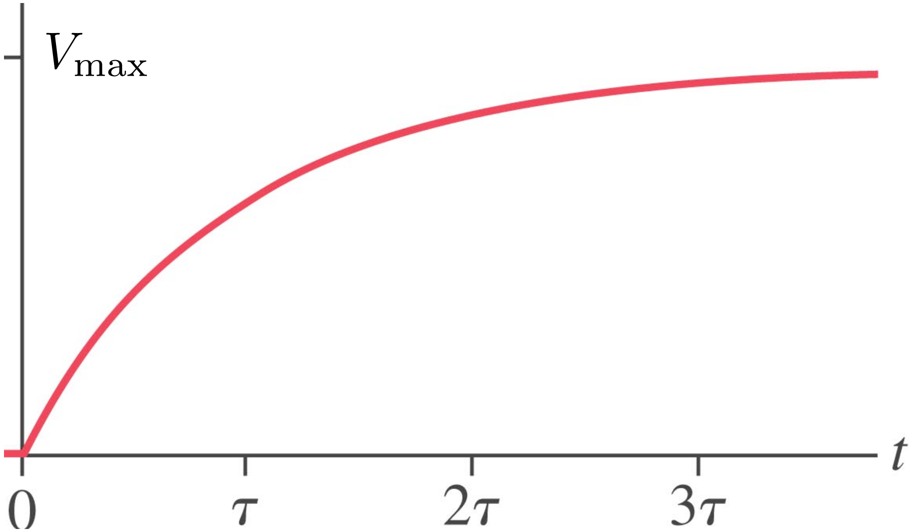
Worksheet Question 8

RC Circuits

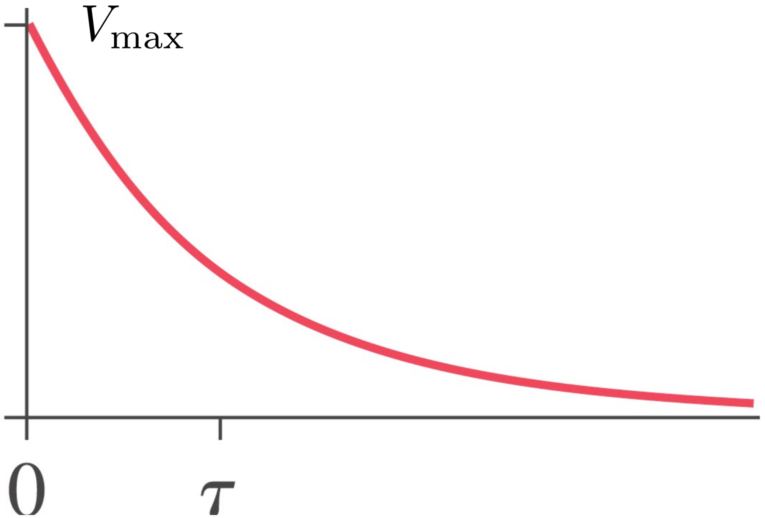
Charging and discharging a capacitor is governed by exponential laws.

charging:



$$V_C = V_{max}\left(1 - e^{-t/RC}\right)$$

discharging:



$$V_C = V_{max}\, e^{-t/RC}$$

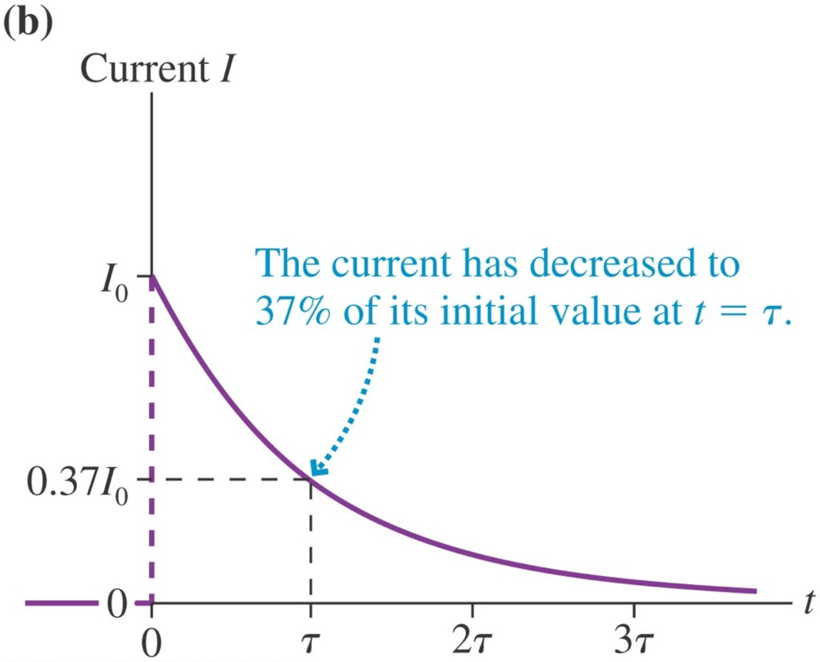
Circuit Construction Kit PhET

RC Circuits

We can define a time constant to characterize the exponential decay

$$\tau = RC$$

It's mathematically identical to the lifetime in radioactive decay.



Capacitors Store Energy

The work required to move a little bit of charge is

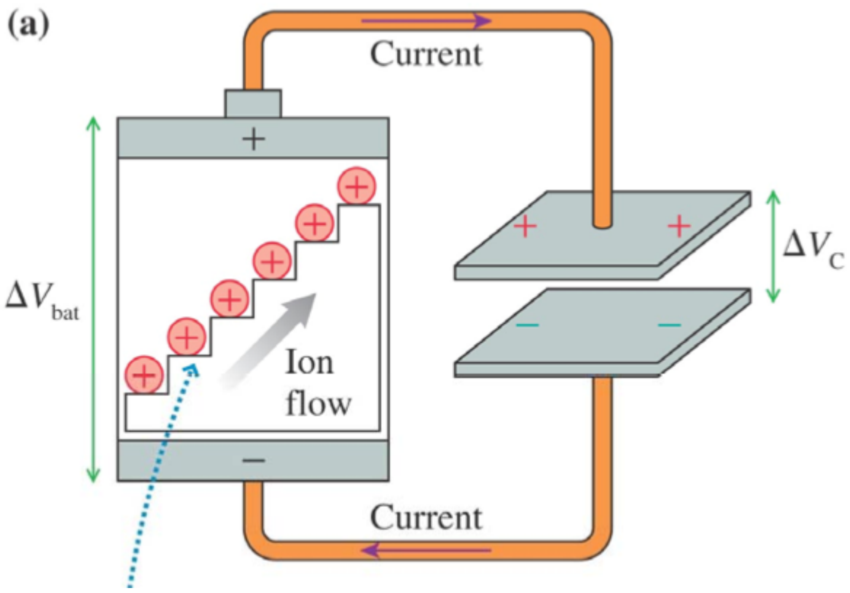
$$dW = dU = V\,dq = \frac{q}{C}dq$$

which gives the integral

$$U = \int_0^Q \frac{q}{C}dq$$

The potential energy in a capacitor is given by.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$



Electric Fields Store Energy

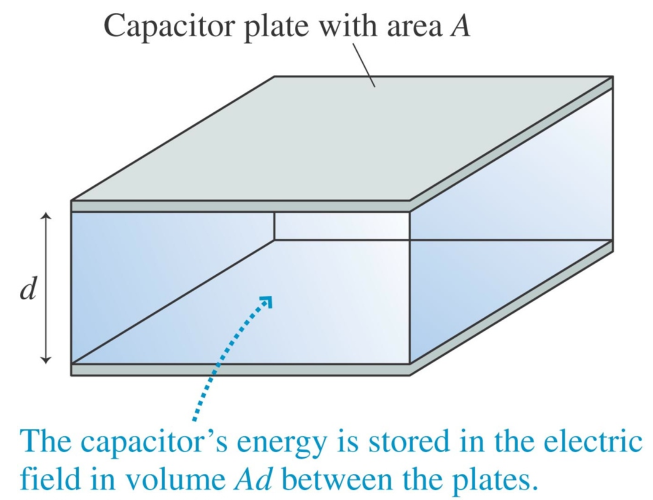
For a parallel plate capacitor we find that

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}(Ed)^2 = \frac{\epsilon_0}{2} \times \text{Volume}$$

The energy density of the electric field is

$$u = \frac{\epsilon_0}{2}E^2$$

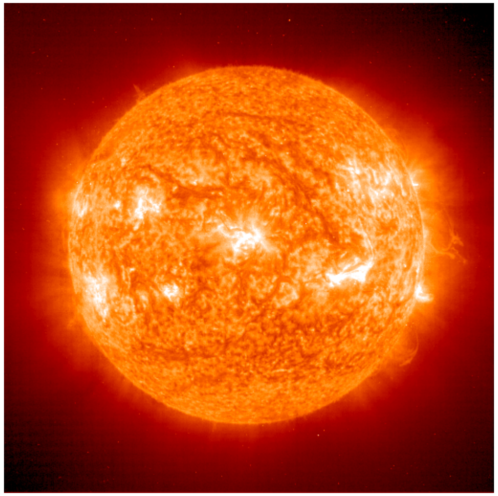
This energy density a **general result**. All electric fields have this energy density.



“Empty Space” Has Energy

But James, the energy is stored in the charge configuration, not the field. You’re playing shell games with us.

The electric field can exist in the absence of charge. The fields wiggle in symbiosis with magnetic fields to make light.



The smallest unit of wiggling electric fields is called the **photon** (discovered separately from radiation).

Electromagnetism

Big Picture

1. **Understand the fundamentals** of electrostatics and magnetism.
   - Coulomb's law, the electric field, electric potential, force on a charge

2. **Use these fundamentals** to "build" circuit components. Learn to analyze these circuits
   - capacitor, current, resistor, loop law, circuit analysis

3. **Express these fundamentals** in the form of Maxwell's equations. See how Maxwell's equations predict something new.
   - Gauss's Law
   - Where are Maxwell's other three equations?

Clicker Question

This graph shows V_C of a capacitor that is separately discharged through three difference resistors.

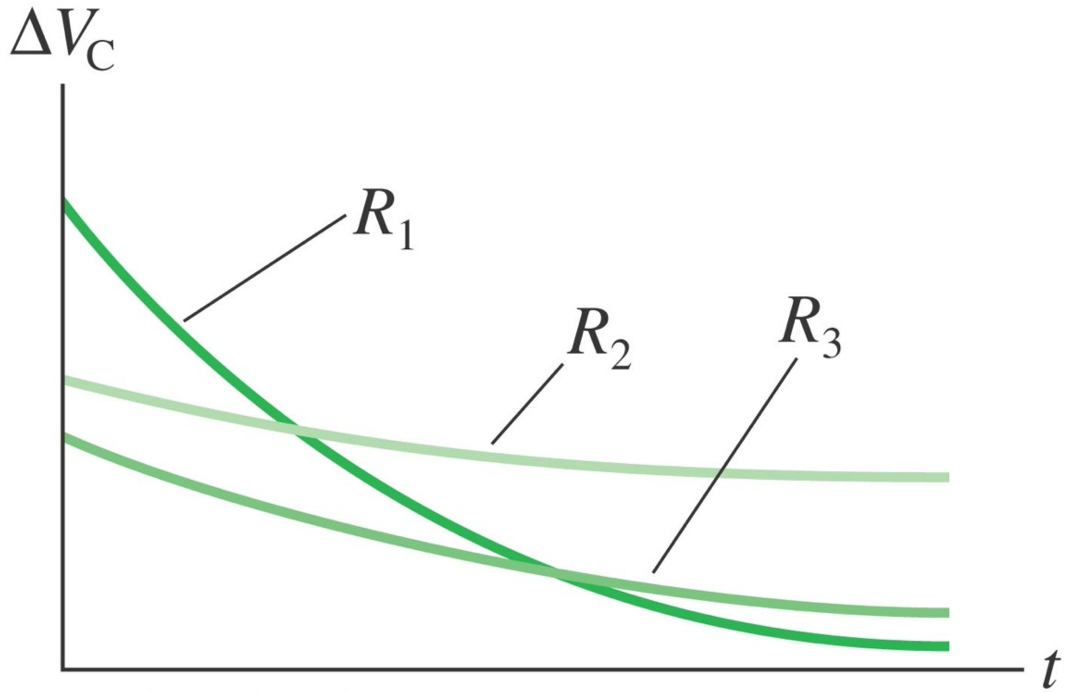
Rank the value of the resistance from smallest to largest.

a) $R_1 > R_2 > R_3$

b) $R_3 > R_2 > R_1$

c) $R_1 > R_3 > R_2$

d) $R_2 > R_3 > R_1$



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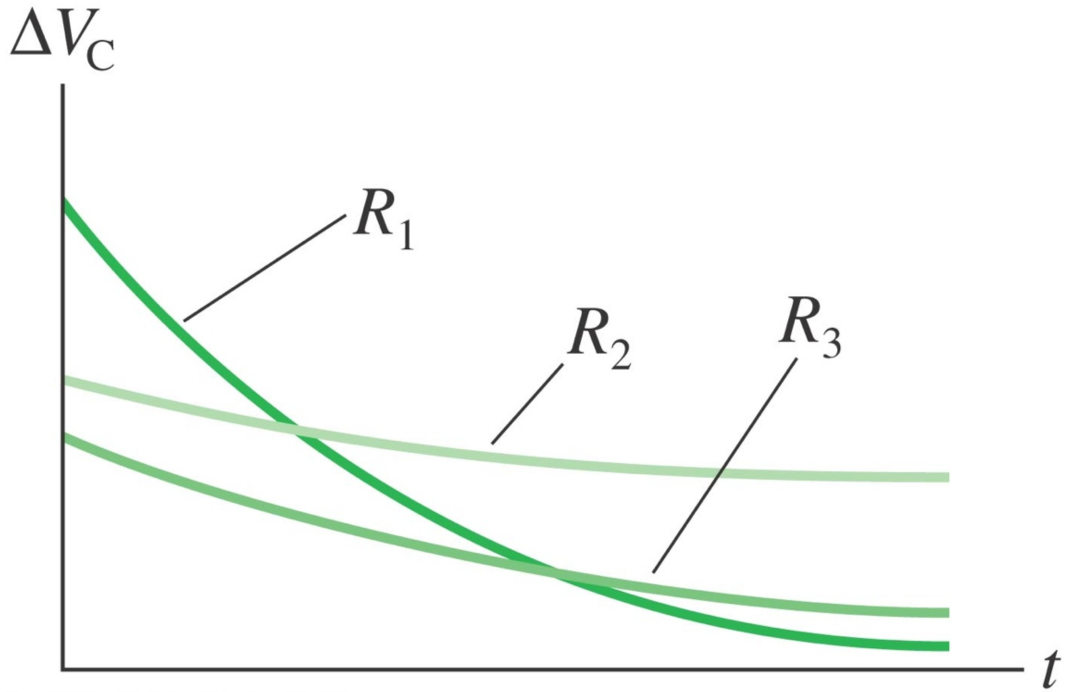

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Supplimentary

Putting a dielectric in a capacitor lowers the electric field, which **increases the capacitance.**

There are two pictures:

1. The **dielectric changes epsilon_0 to epsilon**, which changes how well the electric field can permeate space.

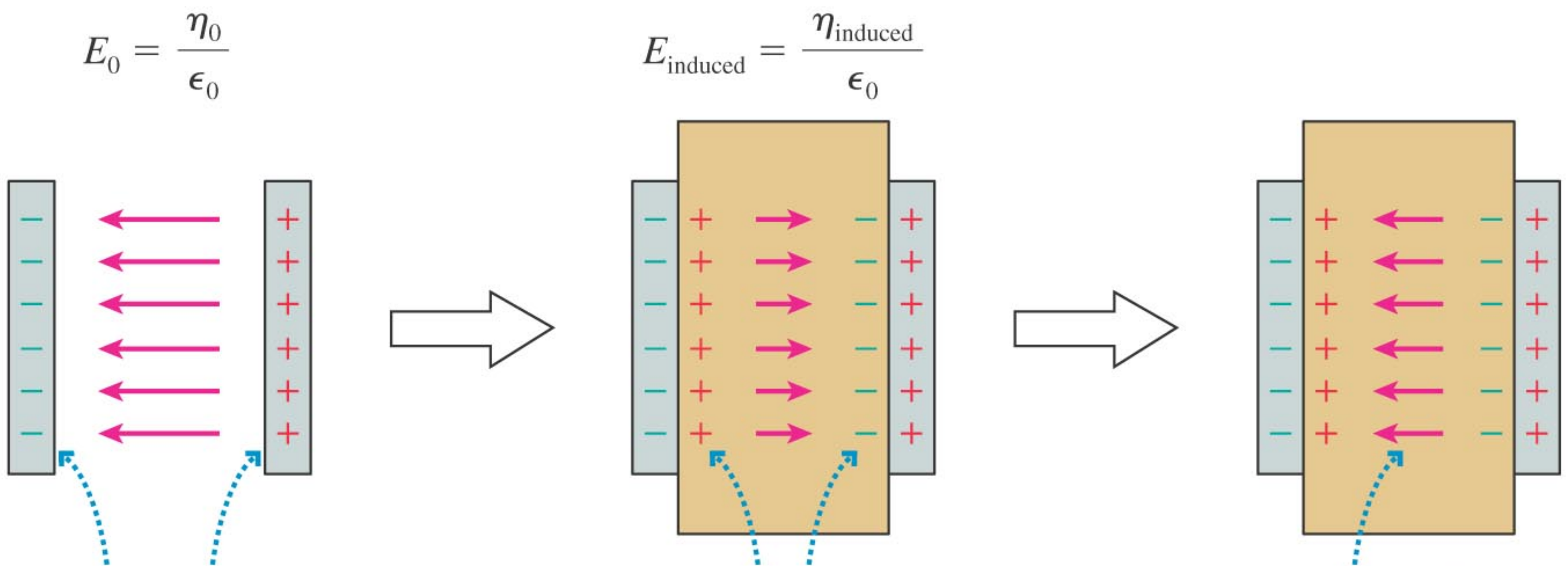
$$E_{\text{without dielectric}} = \frac{\eta}{\epsilon_0} \longrightarrow E_{\text{with dielectric}} = \frac{\eta}{\epsilon} = \frac{\eta}{\kappa \epsilon_0}$$

Because \kappa is positive, E decreases, and the capacitance increases.

2. **Superposition of the electric fields.** The electric field polarizes the dielectric. The polarization actually creates an electric field that opposes the original field, reducing the net field.

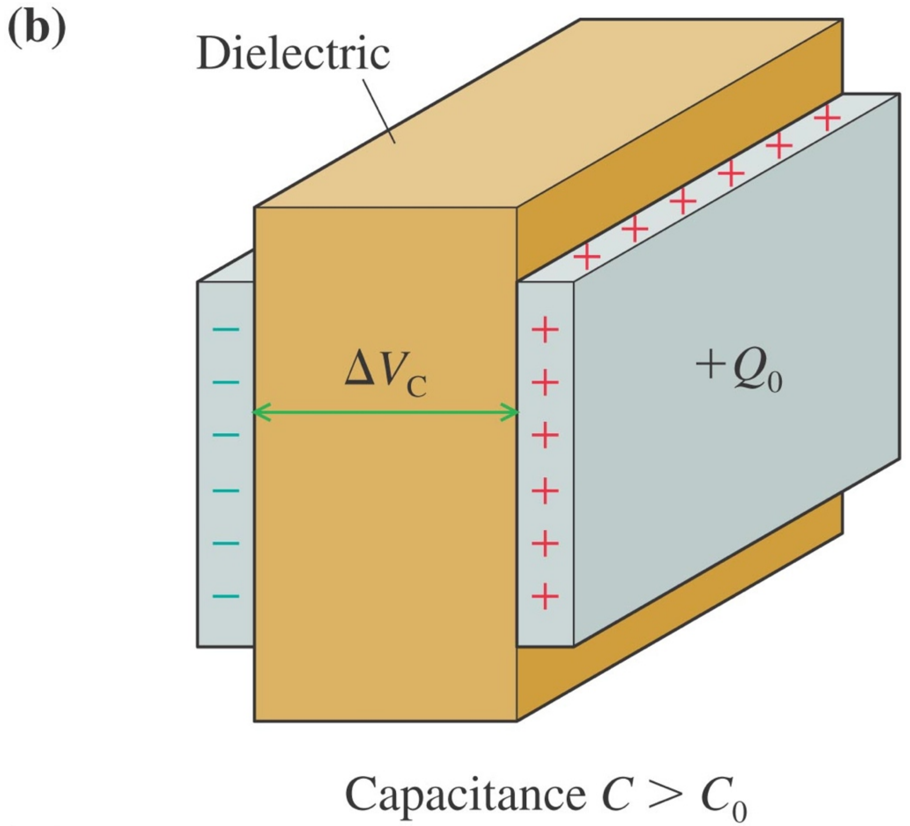
Dielectrics (in tutorial)

The polarization of an insulator in between the plates decreases the net electric field.



Lowering E increases capacitance: $C = \frac{Q}{V} = \frac{Q}{Ed}$

Dielectrics



The capacitance with the dielectric C is **greater than** the capacitance in a vacuum C_0.

The **dielectric constant** kappa κ is the ratio of the new capacitance to the capacitance in a vacuum.

$$C = \kappa C_0$$