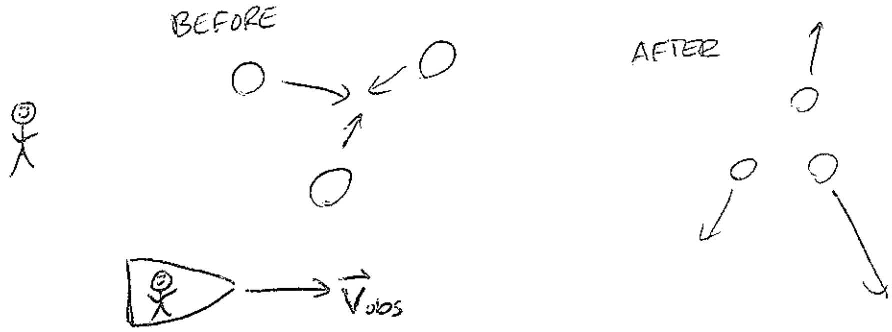
We learned that in any process, the sum of

$$\vec{p} = \gamma m \vec{v}$$

for all objects is the same before and after.



According to the principle of relativity, an observer in another frame of reference will also measure the total momentum to be conserved.

In the observer's frame, each object has a different momentum than in the original frame. If we think about how far the observer sees the object move in some amount of prober time for the object, it will be:

This uses the Lorentz Transformation or the "rulers & clocks" movie

$$\Delta x' = \gamma_{observer}\left(\underset{\substack{\text{distance moved}\\ \text{in original frame}}}{\Delta x} - V_{observer} \cdot \underset{\substack{\text{time passed}\\ \text{in original frame.}}}{\Delta t}\right)$$

So the x-momentum as observed in the new frame is

$$p_x' = m\frac{\Delta x'}{\Delta t_{proper}} = \gamma_{observer}\left(\underbrace{\frac{\Delta x}{\Delta t_{proper}} \cdot m}_{\text{this is } p_x} - m V_{observer} \underset{\text{this is } \gamma}{\frac{\Delta t}{\Delta t_{proper}}}\right)$$

This gives:

$$[p_x'] = \gamma_{\text{observer}}\left([p_x] - \frac{v_{\text{observer}}}{c^2}\cdot\left[\gamma_{\text{object}}\, m\, c^2\right]\right)$$

x momentum observed in new frame

x momentum observed in original frame

relativistic energy observed in original frame.

Now, in any collision, the sum of p_x for all objects is the same before and after the collision.

Also, the sum of p_x' for all objects is the same before and after the collision.

According to the equation at the top, it then follows that the sum of $\gamma_{\text{object}} mc^2$ for all objects is the same before and after every collision.

So momentum conservation plus the principle of relativity (that tells us $\vec{p}$ is conserved in any frame) implies that the relativistic energy $E = \gamma mc^2$ is also conserved.