## PHYSICS WORKSHEET: Prognostication

In this worksheet, you will go through an example of predicting the future positions and velocities of an object given the forces on the object. We'll use the following strategy:


There are many different ways to find a solution of the final equations. We'll discuss some of them in the next few classes. However, we will start with a method that can be used no matter how complicated the system is: we predict the position and velocity at slightly later times using the definition of the derivative:

$$
\begin{aligned}
\vec{r}(t+\varepsilon) & \approx \vec{r}(t)+\varepsilon \cdot \vec{V}(t) \\
\vec{V}(t+\varepsilon) & \approx \vec{V}(t)+\varepsilon \cdot \vec{a}(t)
\end{aligned}
$$

and then repeat this many times to get to the desired time.

Problem 1: A projectile with air drag:
A person throws a ball with initial velocity $\left(v_{x}, v_{y}\right)$. We would like to predict the trajectory of the ball. To do this, we need to know that the drag force acts in the direction opposite to the ball's velocity, and has a magnitude that is some function $F(v)$, where $v$ is the ball's speed. For this question, we will assume that $F(v)=D m v^{2}$, where $m$ is the mass and $D$ is a number.

Step 1: Draw a picture of the ball, showing coordinate axes, the velocity vector and its components.

Step 2: Identify the forces on the ball. Draw another diagram of the ball showing these forces.

$$
\bigcirc
$$

For the next part, it will be useful to break the forces into components:

$$
F_{x}=
$$

$F_{y}=$

Step 3: Write down the components of the equation $\frac{d \vec{v}}{d t}=\frac{1}{m} \vec{F}_{N E T}$ (the right hand sides should only depend on $x, y, v_{x}, v_{y}$, and various constants).

$$
\frac{d v_{x}}{d t}=
$$

$$
\frac{d v_{y}}{d t}=
$$

Also, fill in the right hand sides here:


Step 4: Use these equations to make our predictions.
Fill in the numerical values for the positions and velocities at $t=0$ and the drag coefficient $D$, as measured in class:

$$
\begin{array}{ll}
t=0: & x= \\
& y= \\
& v_{x}= \\
& v_{y}=
\end{array}
$$

$$
\text { coefficient } D=
$$

Using this data, predict $x, y, v_{x}$, and $v_{y}$ at $t=0.01$ s and (if you have time) $t=0.02 \mathrm{~s}$ :

$$
t=0.01 \mathrm{~s}
$$

$$
x=
$$

$$
y=
$$

$$
v_{x}=
$$

$$
V_{y}=
$$

$t=0.025:$

$$
x=
$$

$$
y=
$$

$$
V_{x}=
$$

$$
V_{y}=
$$

Extra: how could we measure the drag coefficient D? HINT: what happens after an object has been falling for a time?

