LAST TIME:

Given $\vec{x}$ and $\vec{v}$ now

$\downarrow$

Use Newton's second law to predict rate of change of velocity: $\dfrac{d\vec{v}}{dt} = \dfrac{1}{m}\vec{F}$ ← this is 3 equations!

Rate of change of position: $\dfrac{d\vec{x}}{dt} = \vec{v}$

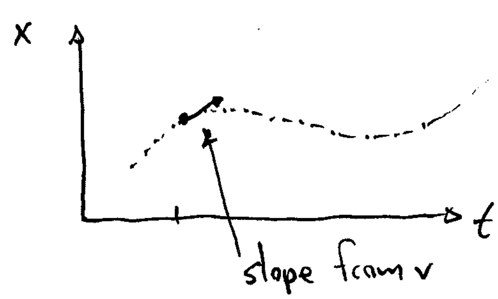
$\downarrow$

Use these rates of change to predict $\vec{x}$ and $\vec{v}$ at a slightly later time:

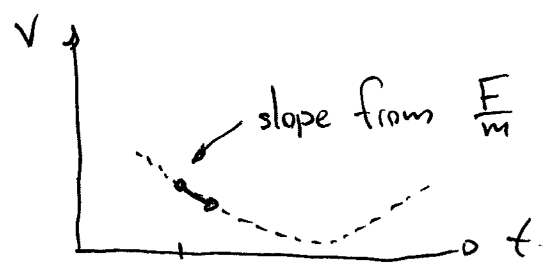
repeat

$$\vec{v}(t+\delta t) \approx \vec{v}(t) + \delta t \cdot \left(\frac{d\vec{v}}{dt}\right)$$

$$\vec{x}(t+\delta t) \approx \vec{x}(t) + \delta t \cdot \left(\frac{d\vec{x}}{dt}\right)$$



Follow the arrow a tiny bit to the next point.

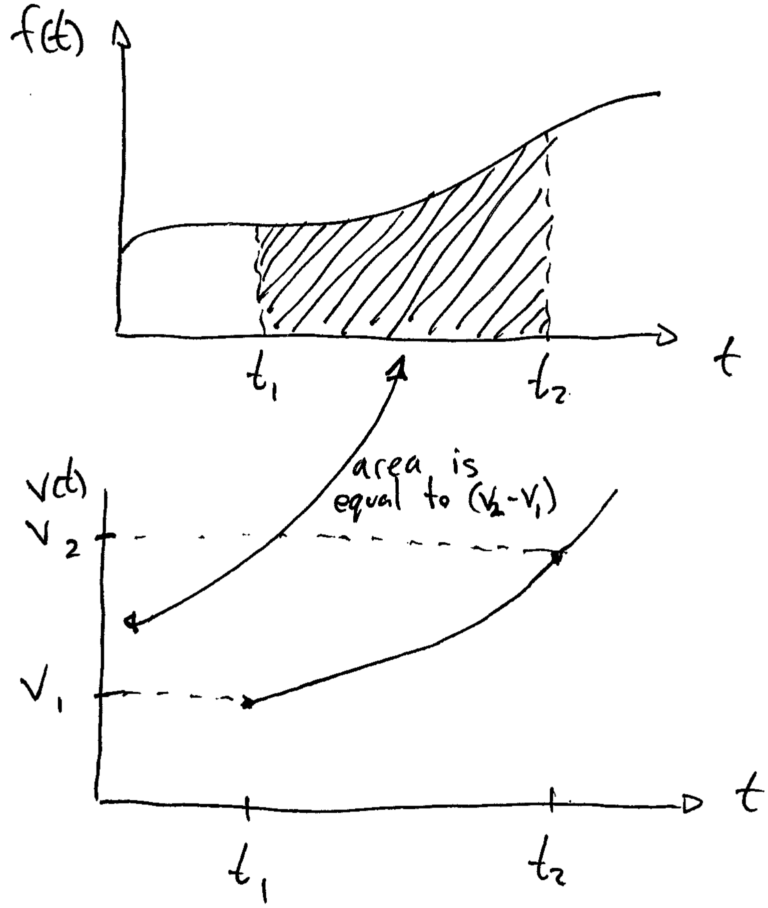


RECAP INTEGRATION:

If $\frac{dv}{dt} = f(t)$ ← this is the acceleration but we're calling it $f(t)$ so it looks like math.

Then $v(t_2) = v(t_1) + \left\{ \text{area under graph of } f(t) \text{ between } t_1 \text{ and } t_2 \right\}$

$= v(t_1) + \int_{t_1}^{t_2} f(t)\, dt$ this is the definition in words.

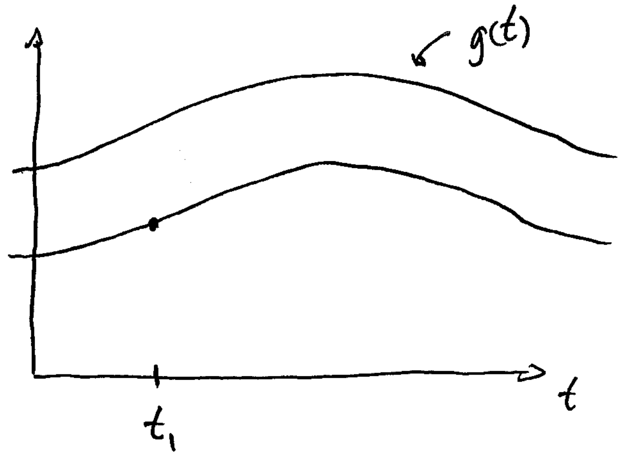


ANOTHER WAY:

Guess a function $g(t)$ such that

$$\frac{dg}{dt} = f .$$

Then $g(t)$ and $v(t)$ have the same time derivative, but differ by a constant.



Given v at some time t we can find the constant.

So
$$v(t) = g(t) + C$$
$$v(t_1) = g(t_1) + C$$

if we know $v(t_1)$ then $C = v(t_1) - g(t_1)$

And

$$v(t) = v(t_1) + [g(t) - g(t_1)]$$

we're given this

we guess g(t) and evaluate this.

SUMMARY:

$$\frac{dv}{dt} = f \quad \rightarrow \quad v(t_2) = v(t_1) + \int_{t_1}^{t_2} f(t)\,dt$$

$$\rightarrow \quad v(t_2) = v(t_1) + \left[g(t_2) - g(t_1) \right]$$

where g is a function such that $\frac{dg}{dt} = f$.

It must be true that:

$$\underbrace{\int_{t_1}^{t_2} f(t)\,dt}_{\text{area under the graph}} = \underbrace{g(t_2) - g(t_1)}_{\text{difference in 2 numbers.}}$$

$\Rightarrow$ Part of the "Fundamental Theorem of Calculus."