LAST TIME:

Conservation of Momentum

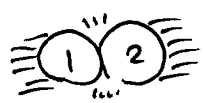
$$\frac{d\vec{p}}{dt} = 0$$

no external influences

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

found using a test object

$$\vec{F}_{1 on 2} = -\vec{F}_{2 on 1}$$



PREDICTING THE FUTURE: let's assume $\vec{p} = m\vec{v}$

"First order differential equations"

$$\frac{d\vec{v}}{dt} = \frac{1}{m}\vec{F}_{net} \qquad \text{also} \qquad \vec{v} = \frac{d\vec{x}}{dt}$$

vector sum of forces.
depends on position, velocity, environment

or

$$\frac{d^2\vec{x}}{dt^2} = \frac{\vec{F}_{net}}{m}$$

"Second order differential equation"

Knowing the force, I can predict the position and velocity. Physics (a large part of it) involves figuring out what these forces are and how they act.

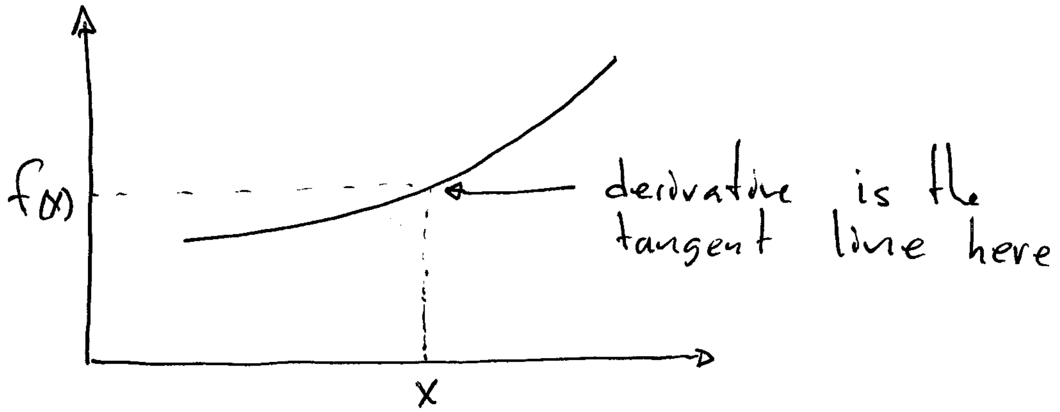
Ways to solve:

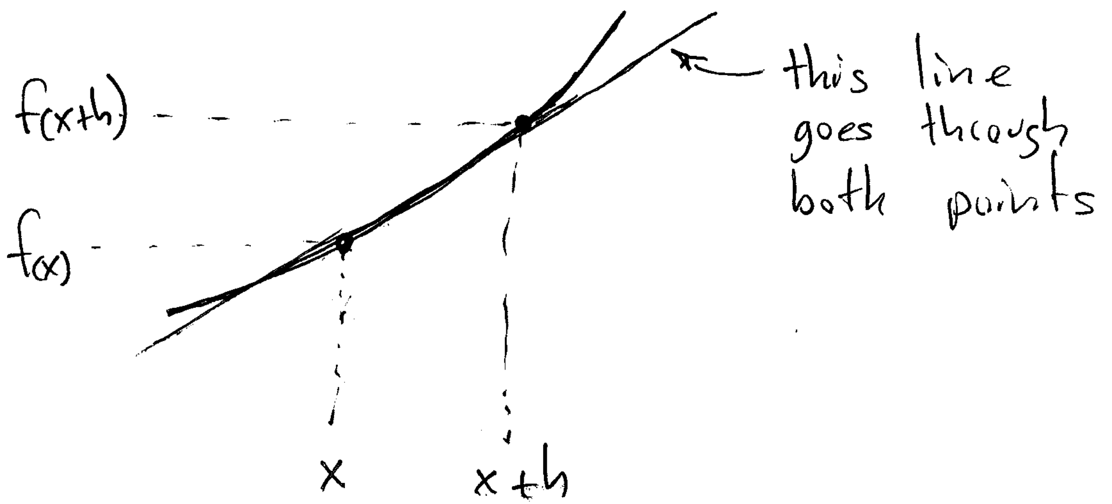
1. Numerical e.g. "Euler Method"

2. (Simple cases) Directly find a function that satisfies equations or guess and check. Math (215)

3. (Even simpler cases) When forces are some known function of time (i.e. don't depend on position or velocity) use <u>integration</u>.

LIMIT DEFINITION OF THE DERIVATIVE:





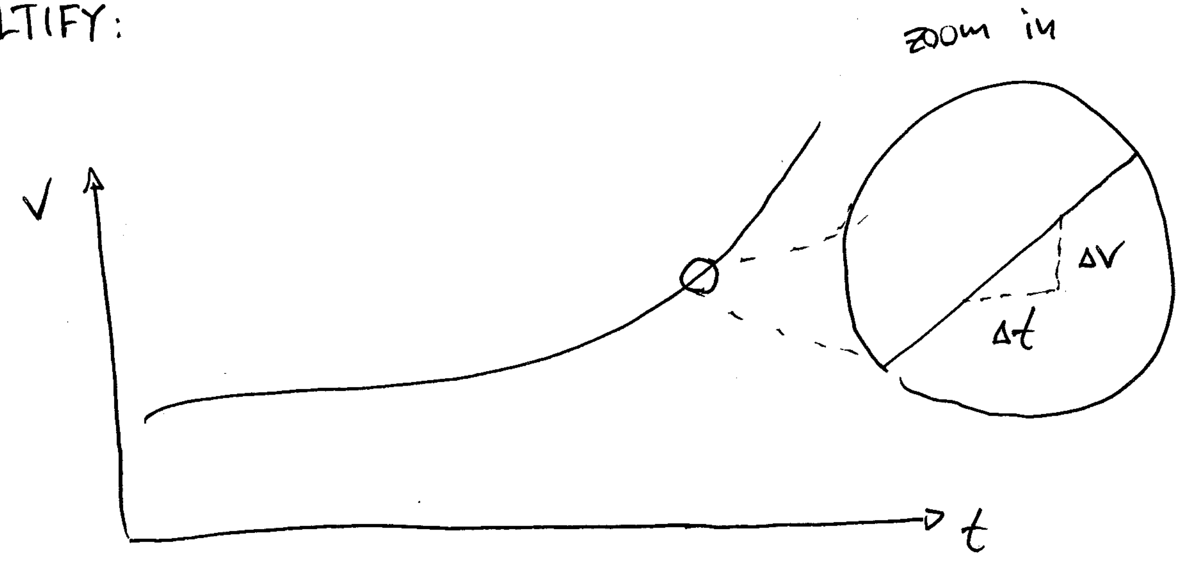
The slope is

$$= \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

This becomes the tangent line when $h \to 0$

$$\frac{df(x)}{dx} \equiv \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

FROM BOLTIFY:



$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} \equiv \frac{d\vec{v}}{dt}$$ derivative of velocity.

Used this technique to find a.

What if we know $\vec{a}$?

$$\frac{\vec{F}_{net}}{m} = \vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \lim_{\delta t \to 0} \frac{\vec{v}(t+\delta t) - \vec{v}(t)}{\delta t}$$

take δt very small, but finite.

$$\frac{\vec{F}_{net}}{m} \approx \frac{\vec{v}(t+\delta t) - \vec{v}(t)}{\delta t}$$

$$\Rightarrow \underbrace{\vec{v}(t+\delta t)}_{\text{velocity in the future!}} = \underbrace{v(t)}_{\text{velocity now}} + \underbrace{\delta t}_{\text{tiny time step.}} \underbrace{\overbrace{\frac{\vec{F}_{net}}{m}}^{\vec{a}(t)}}_{\text{accelleration now.}}$$

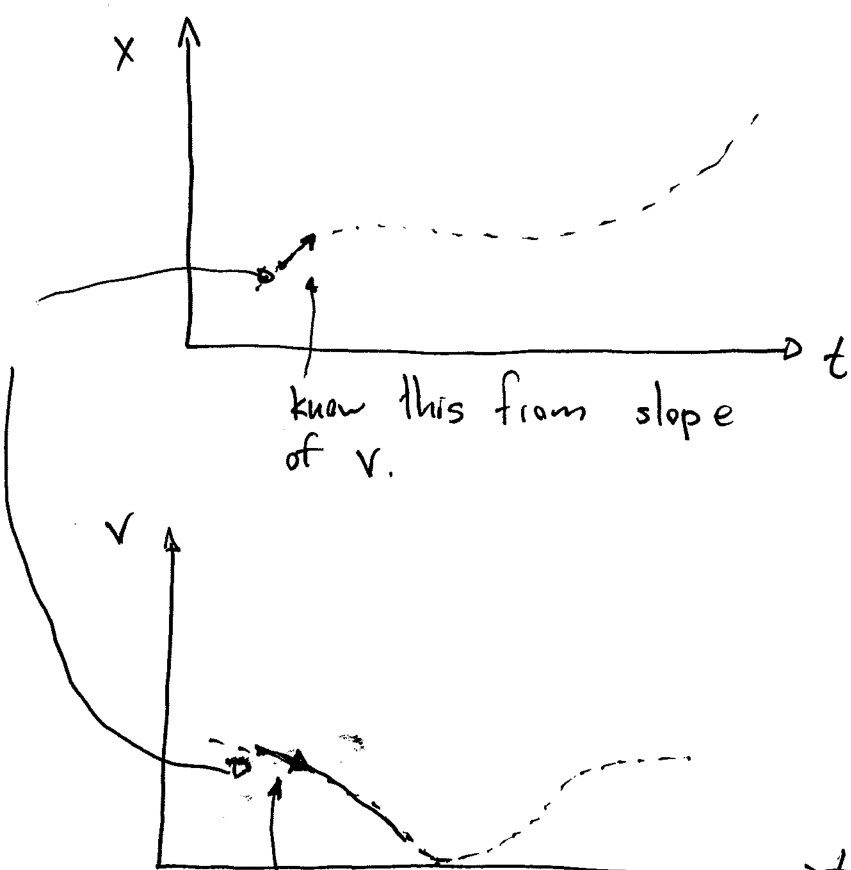
also: Since $\frac{dx}{dt} = v$ we get

$$x(t+\delta t) \approx x(t) + \delta t \, v(t)$$

We have 2 equations that given the ~~force~~ position and velocity now, ~~whi~~ with a force, we get the velocity and position in the future!

Then repeat!

only need x_0 and v_0 to get all of the future!



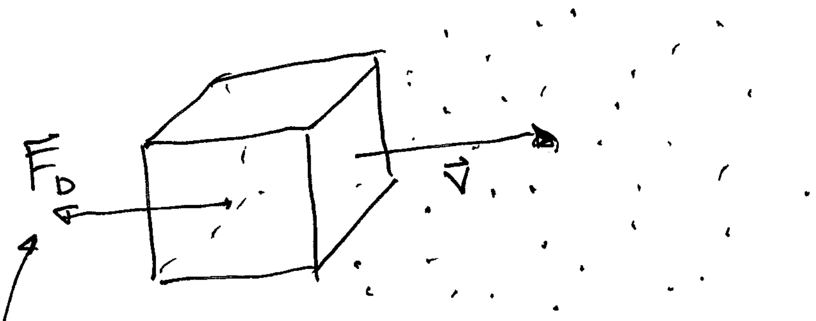
know this from slope of v.

Follow the arrow a bit to get the next point.

Figure out this slope from $\frac{1}{m} F$.

AIR DRAG:

Imagine a thing moving through the air.



depends on v, A, ρ

$$[v] = m/s \quad \text{or} \quad \frac{L}{T}$$

$$[A] = L^2$$

$$[\rho] = \frac{M}{L^3}$$

Need $[F]$ = Newtons $= \frac{ML}{T^2}$

$$F \propto v^2 \rho A = \frac{1}{2} C_D \rho A v^2$$

$$[F] = \frac{\cancel{L^2}}{T^2} \frac{M}{\cancel{L^3}} L^2 = \frac{ML}{T^2}$$