LAST TIME:

Momentum conservation $\xrightarrow{\text{isolated object}}$ Newton's 1st Law

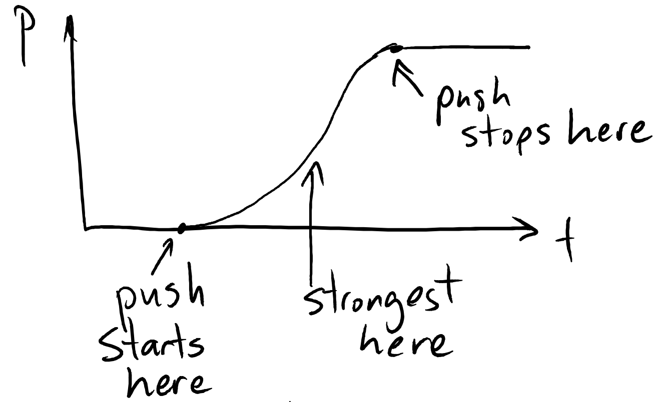
$\downarrow$

★ **two objects that experience the same external influence will experience the same change in momentum**

$M_1 v_1 = M_2 v_2$

We can quantify the strength of an external influence by the change in momentum it produces:

e.g. pushing an object initially at rest:



Can define FORCE by $\frac{dp}{dt}$ on some test object.

e.g. Force in Newtons = change in momentum per second of 1kg space-salmon

- Now apply this force to any other object for some time dt.
- Change in momentum dp will be SAME as for the test object (by result * above)

So: $$\boxed{\frac{d\vec{p}}{dt} = \vec{F}_{NET}}$$ for ANY object

This is NEWTON'S 2ND LAW

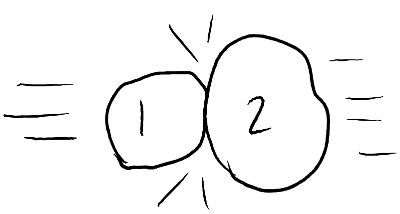
small speeds: $\vec{p} = m\vec{v}$

$v << c$

speed of light

so $\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$

Also: for interaction between 2 objects,



In time Δt, total $\vec{p}$ conserved so

$$\Delta \vec{p}_1 = -\Delta \vec{p}_2$$

$$\Rightarrow \frac{\Delta \vec{p}_1}{\Delta t} = -\frac{\Delta \vec{p}_2}{\Delta t}$$

Conclusion:

$$\boxed{\vec{F}_1 = -\vec{F}_2}$$

force of 2 on 1

force of 1 on 2

Newton's 3rd Law

NOT always true of NET force on 1 + 2 since could be other forces.

PREDICTING THE FUTURE

$$\frac{d\vec{v}}{dt} = \frac{1}{m}\vec{F}_{NET}$$

vector sum of forces on object

depends on position, velocity, environment of object

e.g. gravity

friction

push/pull

normal

air drag

⋮

Assuming we know forces for all possible positions + velocities of object, can predict $\vec{r}(t)$, $\vec{v}(t)$ from $\vec{r}(t=0)$ and $\vec{v}(t=0)$

key point: $\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{v(t+\Delta t) - v(t)}{\Delta t} = \frac{1}{m} F_{NET}$

for $\Delta t = \varepsilon$ (v. small)

approx $\stackrel{=}{}$

$$\frac{v(t+\varepsilon) - v(t)}{\varepsilon} \approx \frac{1}{m} F_{NET}$$

$$v(t+\varepsilon) \approx v(t) + \varepsilon \cdot \frac{1}{m} F_{NET}$$

velocity at slightly later time

velocity now

depends on position + velocity now

Also: $x(t+\varepsilon) \approx x(t) + \varepsilon \cdot v(t)$ since $\frac{dx}{dt} = v$

Given x, v now, get x, v at a slightly later time. Repeat!

More + more precise as $\varepsilon \to 0$.