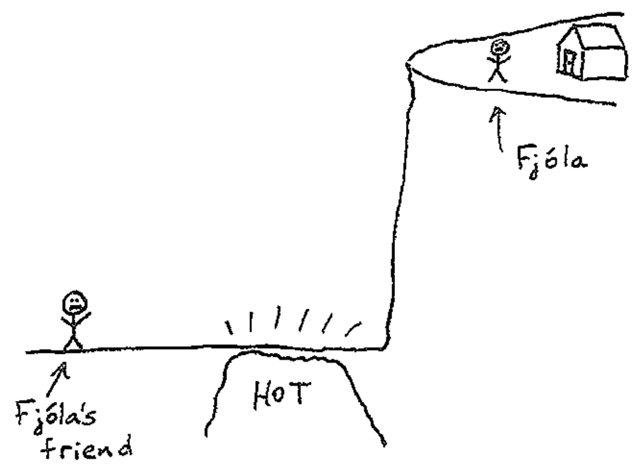
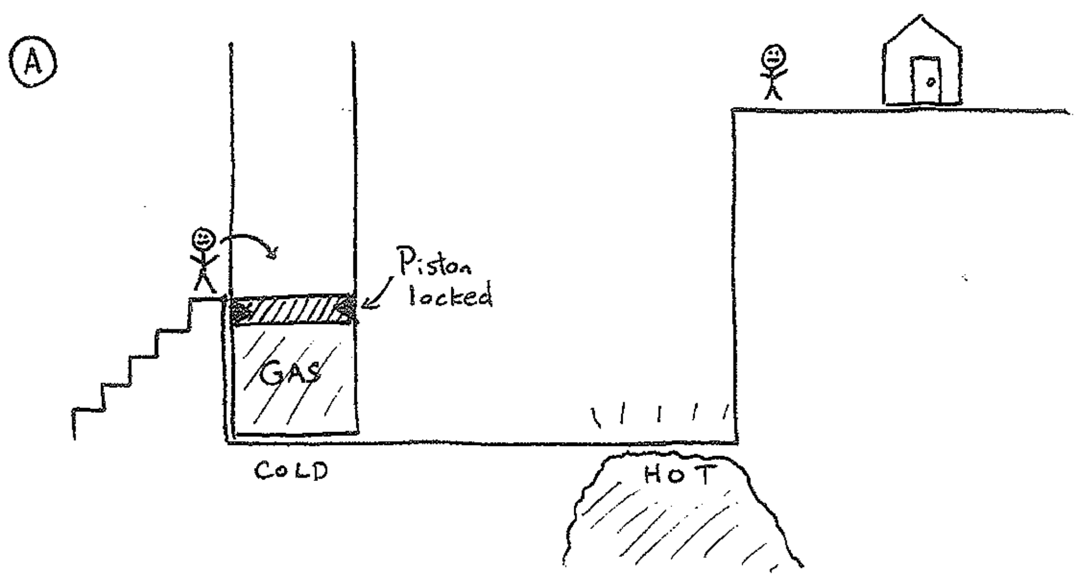
SOLUTIONS

Physics Tutorial: The Gas-O-Vator

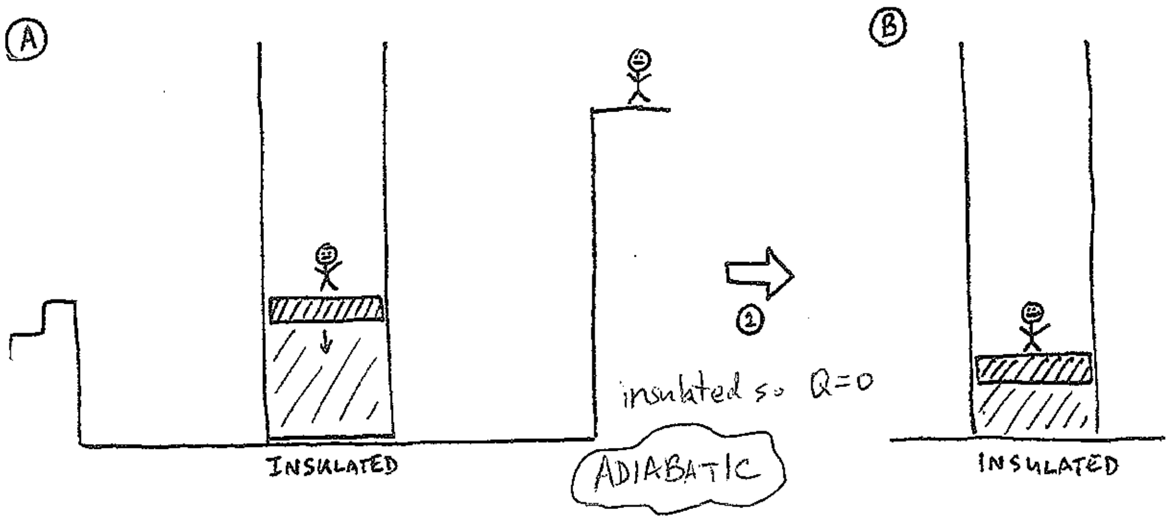
Fjóla lives in Iceland in a small house at the top of a medium-sized cliff above a large bed of very hot rock heated by underground volcanic activity. She is unhappy because her friends do not like to come visit her. One day, in the self-help section of the local bookstore, she sees a book called "The Power of Heat." It turns out to be a misplaced book about thermodynamics, but she reads it anyways, and it gives her an idea. Fjóla decides to build a



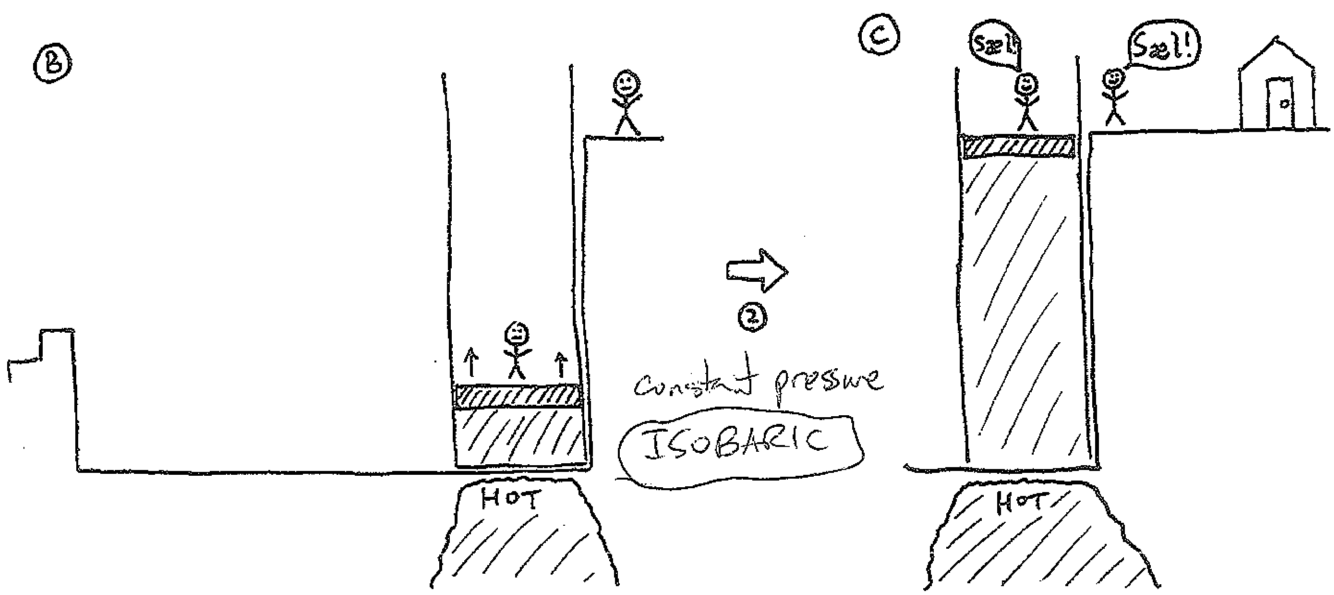
heat-powered elevator to make it easier for her friends to visit. The elevator consists of a moveable column of gas with a piston on top large enough for a person to stand on. The column initially sits on a patch of cold ground. The piston is locked in place, so that Fjóla's friend can step on to the piston from a platform:



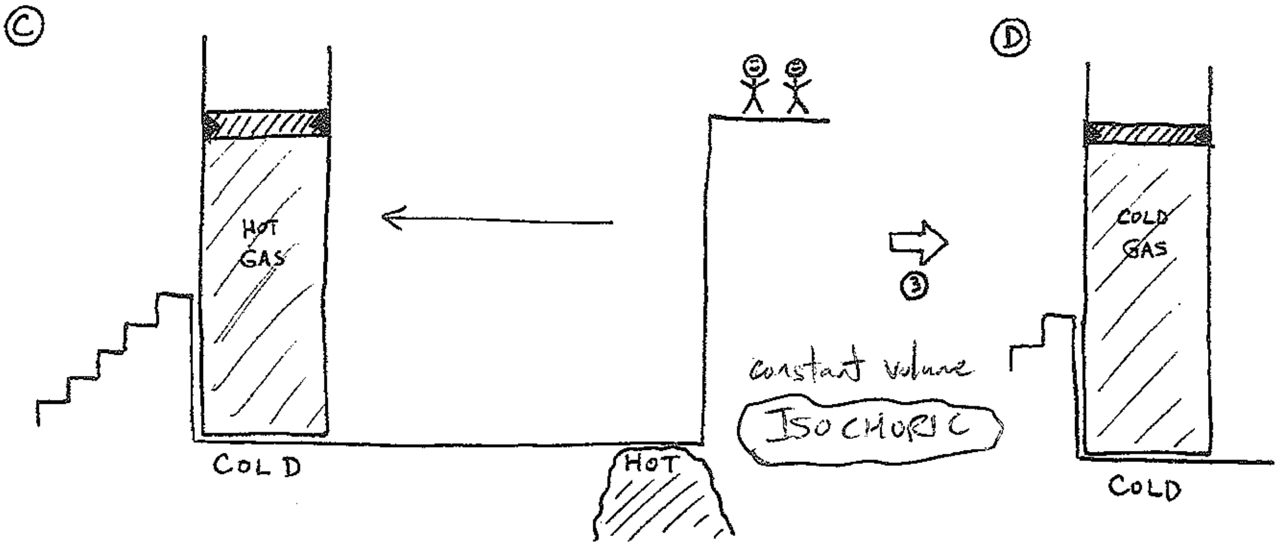
The column is then moved (it's on wheels with nearly frictionless bearings so only a negligible amount of work is required to move it) towards the hot ground. In between, the ground is covered in vegetation that provides almost perfect insulation between the column and the ground. The piston is unlocked during this part and the gas compresses:



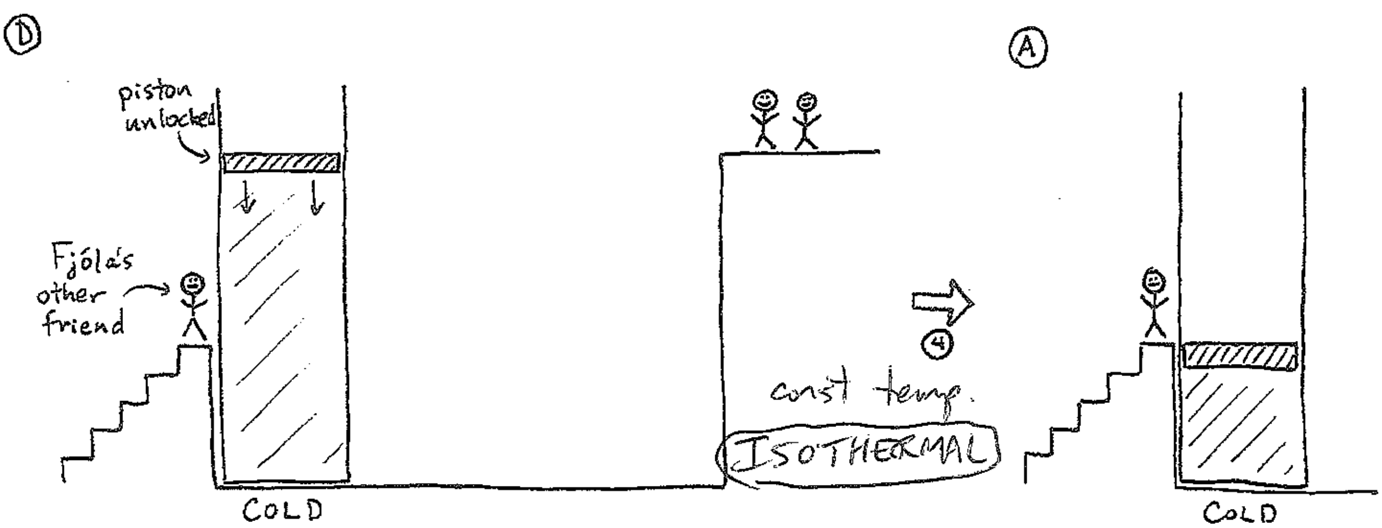
The column then reaches the hot ground, and the gas expands, pushing Fjóla's friend up to Fjóla:



The piston is then locked, and the column pushed back to the cold ground, where the gas cools:



Finally, the piston is unlocked and the weight of the piston compresses the gas back to its original position, ready for Fjóla's next friend to get on: * this is done in a controlled manner so that the gas remains in equilibrium with the cold ground *



QUESTION 1: Label the processes ① - ④ as either isothermal, isochoric, isobaric, or adiabatic.

Question 2: The hot rock below Fjóla's cliff has a temperature of 1350K. If the cliff is 20m high, the atmospheric pressure is 100kPa, the piston has an area of $0.1m^2$ and a mass of 50kg, and if Fjóla's friends each have a mass of 50kg, how many moles of gas does Fjóla need to put in her column?

Looking at (c): $P = P_{atm} + mg/A$ ← piston + friend

$= 100\,kPa + \frac{100kg \cdot 9.8m/s^2}{0.1m^2}$

$= 109.8\,kPa$

$V = 0.1m^2 \times 20m = 2\,m^3$

$T = 1350K$

$\therefore \quad n = \frac{PV}{RT} = \frac{109.8kPa \times 2m^3}{8.31J/K\,mol \cdot 1350K} = 19.6\,mol$

Question 3: If the cold ground has a temperature of 270K, how tall should Fjóla build the platform for her friends to get on the elevator?

Looking at (A):

$P = P_{atm} + \frac{mg}{A}$ ← just piston

$= 100kPa + \frac{50kg \times 9.8m/s^2}{0.1m^2}$

$= 104.9\,kPa$

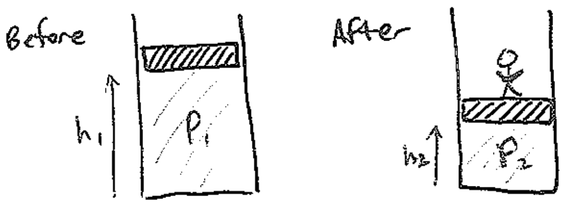
$V = H \cdot 0.1m^2$ H = unknown height.

$T = 270K$

so: $V = \frac{nRT}{P} = 0.419\,m^3$

$\Rightarrow H = \frac{V}{0.1m^3} = 4.19m$

Question 4: After the compression of step 1, what is the final temperature of the gas and the final height of the piston, assuming that the gas is helium? (*Hint: what does conservation of energy tell us?*)



We have $Q=0$ so

$\Delta E = W$

approximate by the grav. force + force from external air pressure

$\Rightarrow n \cdot c_v \cdot \Delta T = -F \Delta h$

$\Rightarrow n \cdot c_v \cdot (T_f - T_i) = (P_{atm} \cdot A + 100\,kg \cdot g)(h_i - h_f)$

$\Rightarrow 244.3\ J/K \cdot T_f + 1.01 \times 10^4\ J/m \cdot h_f = 1.083 \times 10^5\ J$

↑ equation ①

We also have $PV = nRT$ after the compression, so:

$$\left(P_{atm} + \frac{100\,kg \cdot g}{A}\right)(A \cdot h_f) = nRT_f$$

$$\Rightarrow 1.01 \times 10^4\ J/m \cdot h_f = 162.9\ J/K \cdot T_2$$

$$\Rightarrow h_f = 0.0148\ m/K \cdot T_2 \quad ②$$

Plugging in to equation ① and solving for T_2, we get:

$$T_2 = 274.7\,K$$

Plugging back into ② gives:

$$h_f = 4.08\,m.$$

271.12 4.373

Question 5: Looking to reduce a crippling deficit, the Icelandic government decides to tax geothermal energy use. They charge 1 króna per kJ of energy extracted from the ground. How many krónur will it cost Fjóla each time a friend comes to visit? *(Hint: only process 2 is relevant for this part)*

For this part, we have:

$$Q = \Delta E - W$$

$$= n C_v \Delta T \quad + P\Delta V$$

use $C_v = \frac{3}{2}R$

$$= (19.6\,\text{mol}) \cdot \frac{3}{2}\left(8.31\,J/(\text{mol}\,K)\right) \cdot (1350K - 274.7K)$$

$$+ \left(100kPa + \frac{100kg \cdot g}{A}\right) \cdot A \cdot (20m - 4.08m)$$

$$\simeq 4.378 \times 10^5\ J$$

So the cost is **438 króna.** ← about \$3.80 Canadian.

Or we can use

$$Q = n\,C_p\,\Delta T \quad \text{where } C_p = C_v + R = \frac{5}{2}R.$$

to get the same result.

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