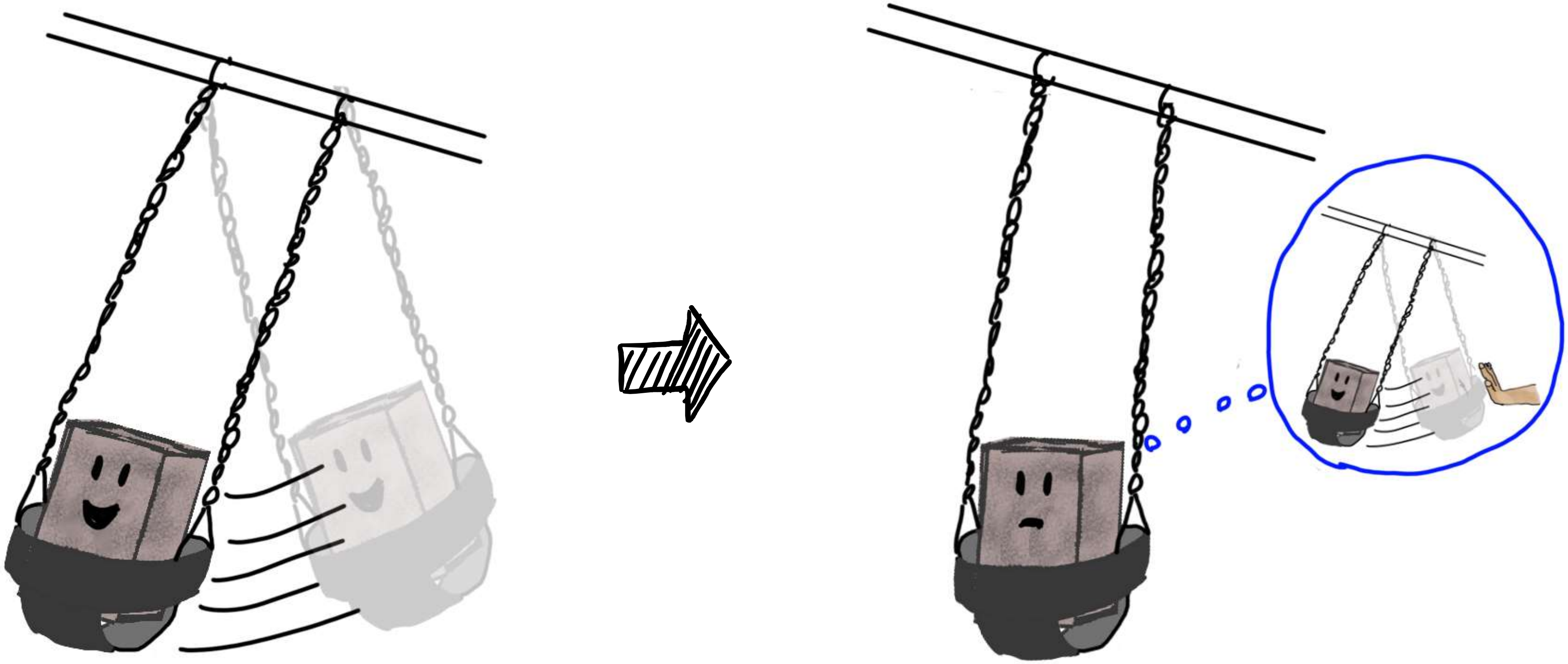
Last time in Phys 157...

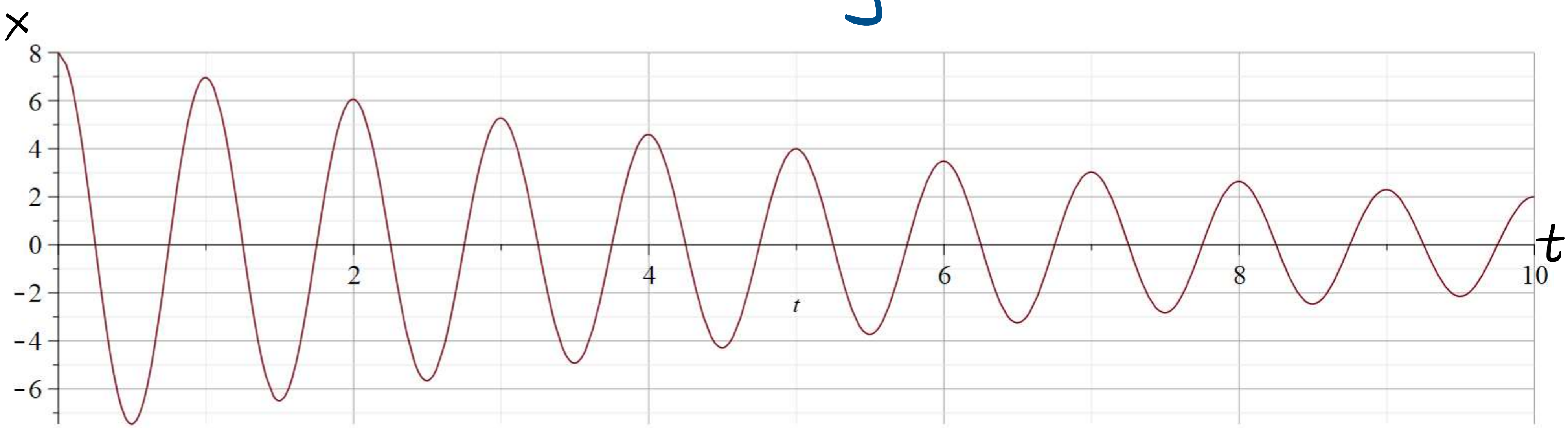


Real oscillators: energy is lost

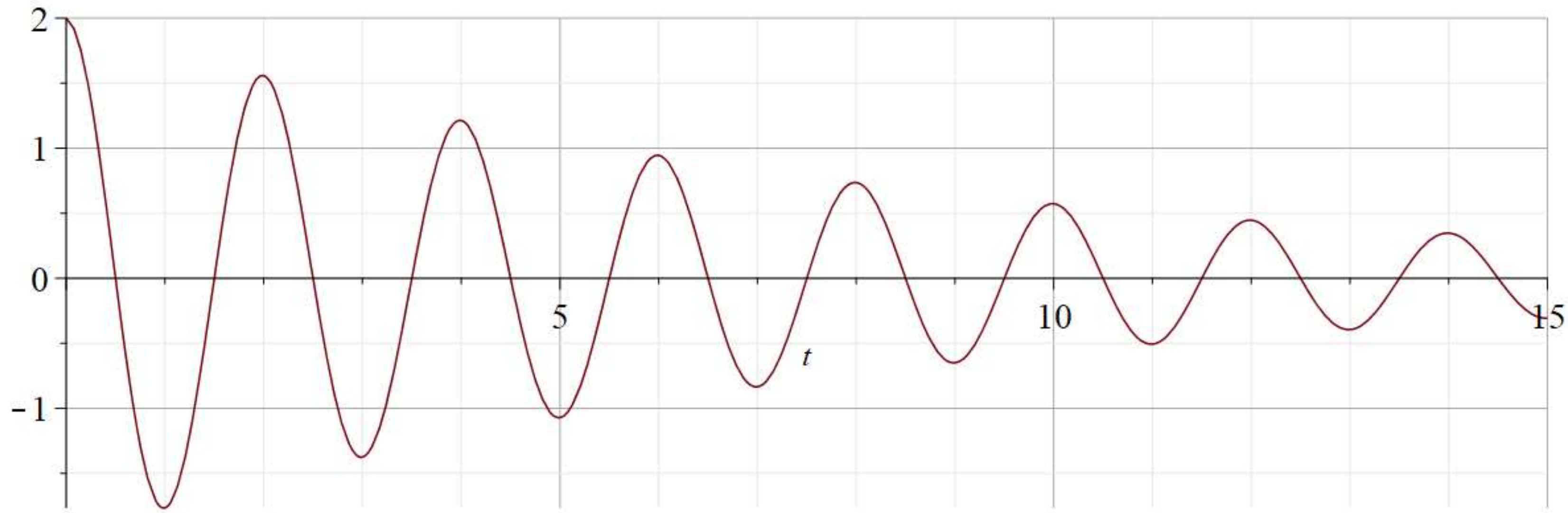
friction

air/fluid drag

heating of system



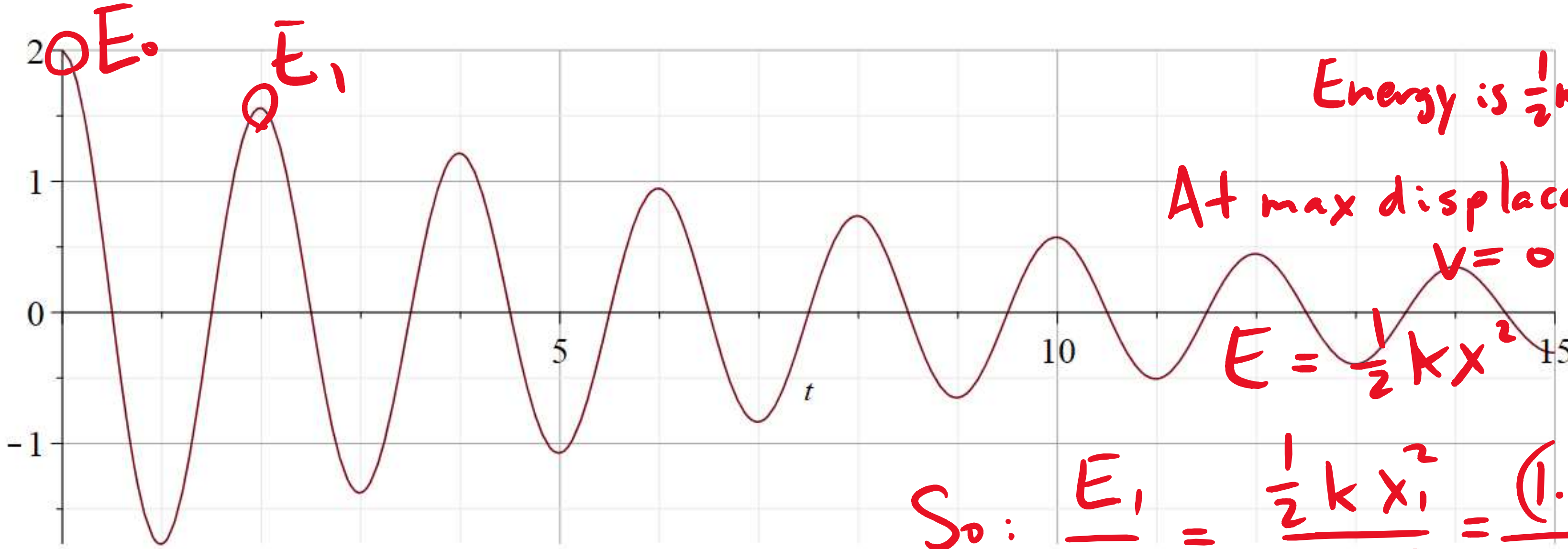
amplitude decreases with time



An object with mass 2kg oscillates on a spring with a small amount of damping.

Roughly what fraction of the energy is lost in one complete oscillation?

A) 6% B) 12% C) 23% D) 40% E) 72%



Energy is $\frac{1}{2}mv^2 + \frac{1}{2}kx^2$

At max displacement, $v = 0$ so:

$E = \frac{1}{2}kx^2$

So: $\frac{E_1}{E_0} = \frac{\frac{1}{2}kx_1^2}{\frac{1}{2}kx_0^2} = \frac{(1.55)^2}{2^2} \approx 0.6$

so 40% has been lost

An object with mass 2kg oscillates on a spring with a small amount of damping.

Roughly what fraction of the energy is lost in one complete oscillation?

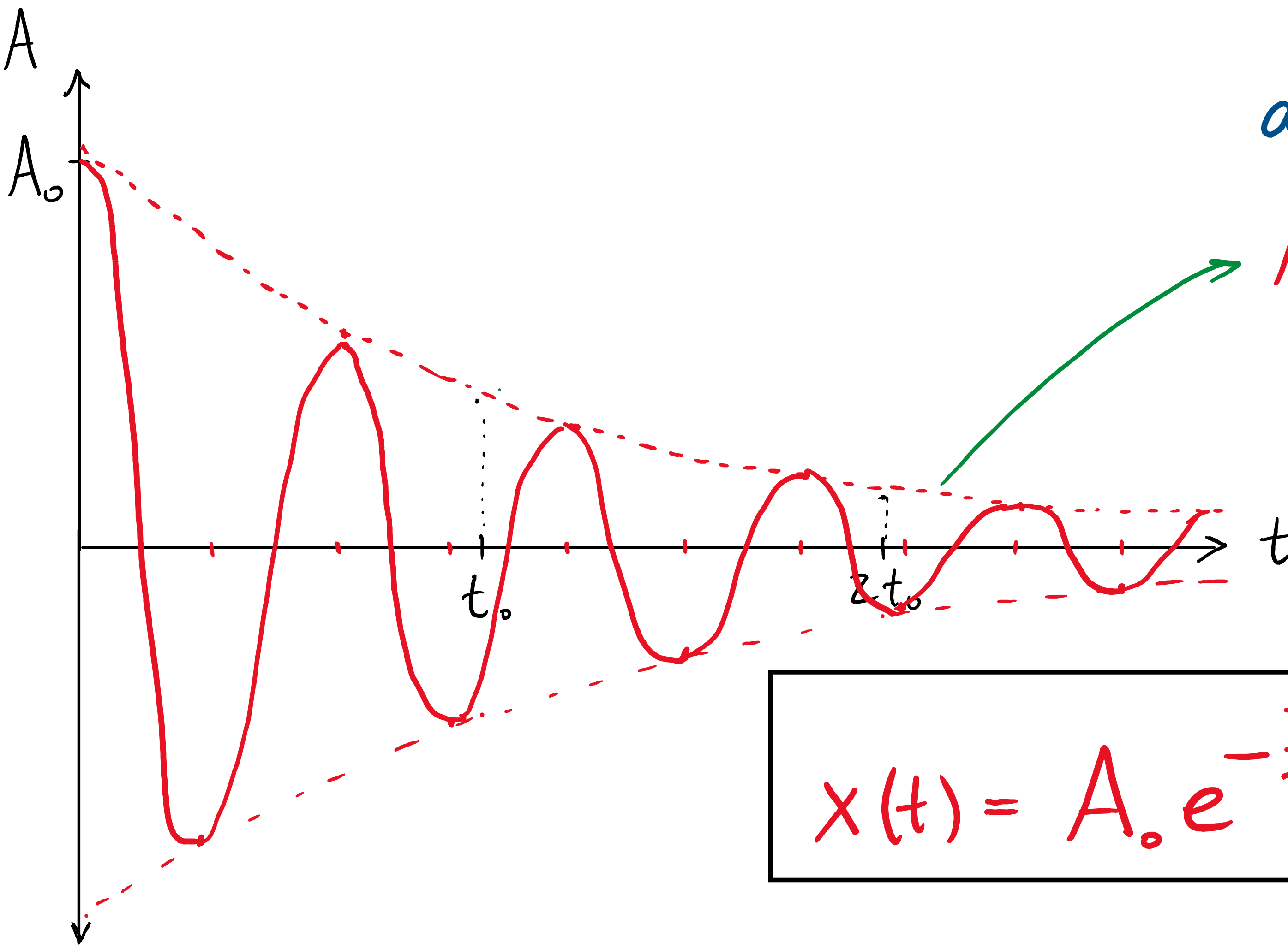
A) 6% B) 12% C) 23% D) 40% E) 72%

Q for an oscillator "quality factor"

$$Q = \frac{2\pi}{\text{fraction of energy lost/cycle}}$$

large Q $\longrightarrow$ small damping

Damped oscillations (if A drops by same fraction each cycle)

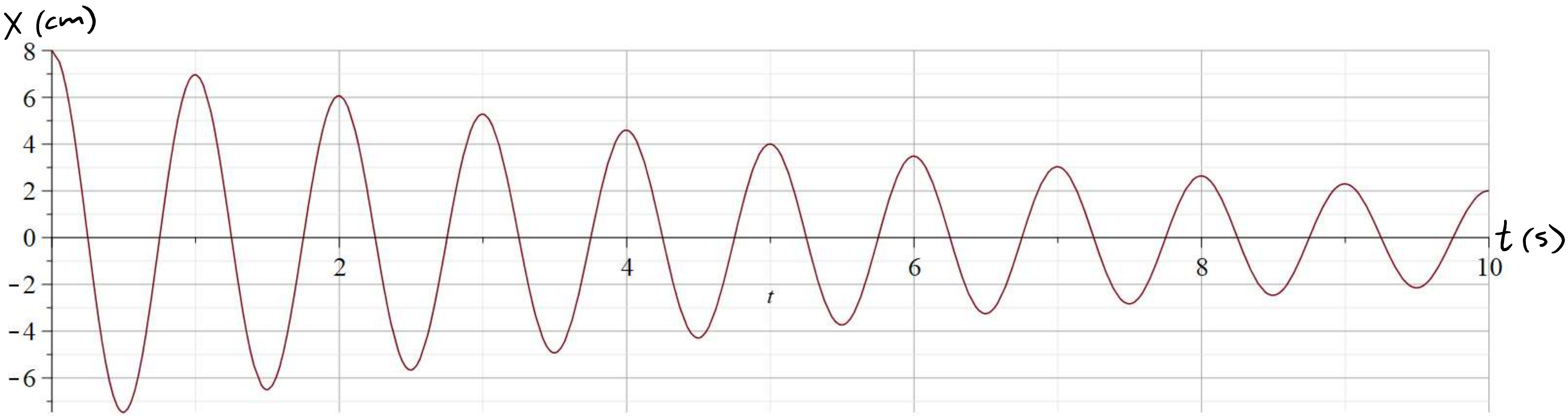


amplitude:

$$A = A_0 e^{-\frac{t}{t_0}}$$

multiplies cosine

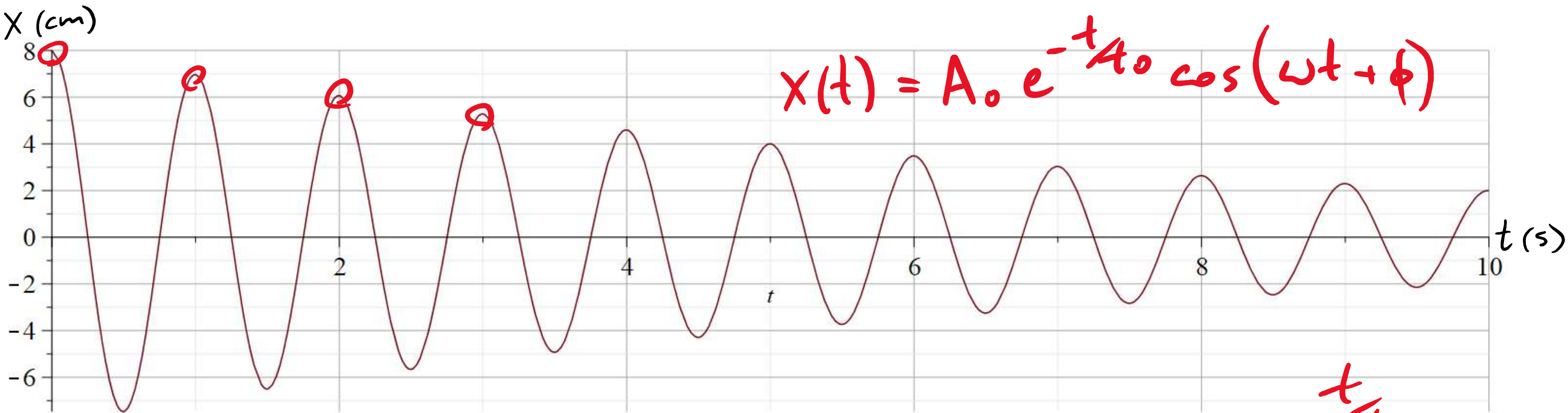
$$\boxed{x(t) = A_0 e^{-\frac{t}{t_0}} \cos(\omega t + \phi)}$$



The graph shows displacement vs time for a damped oscillation. The time constant t_0 in this case is nearest to

A) 1s B) 3s C) 5s D) 7s E)9s

EXTRA: Can you find t_0 exactly given that $x(5s) = 4cm$?



$$x(t) = A_0 e^{-t/t_0} \cos(\omega t + \phi)$$

At circled points, $\cos = 1$ so $x(t) = A_0 e^{-t/t_0}$

The graph shows displacement vs time for a damped oscillation. The time constant t_0 in this case is nearest to

At $t=0$, $x = 8\text{cm}$. At $t = 2s$, $x = 6\text{cm}$.

A) 1s B) 3s C) 5s D) 7s E)9s

$$6\text{cm} = 8\text{cm} \times e^{-\frac{(2s)}{t_0}}$$

$$\Rightarrow e^{-2s/t_0} = 0.75$$

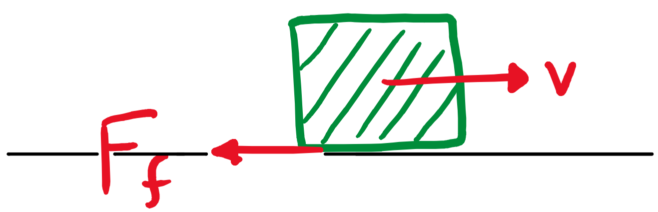
$$\rightarrow -\frac{2s}{t_0} = \ln(0.75)$$

$$\rightarrow t_0 \approx 7s$$

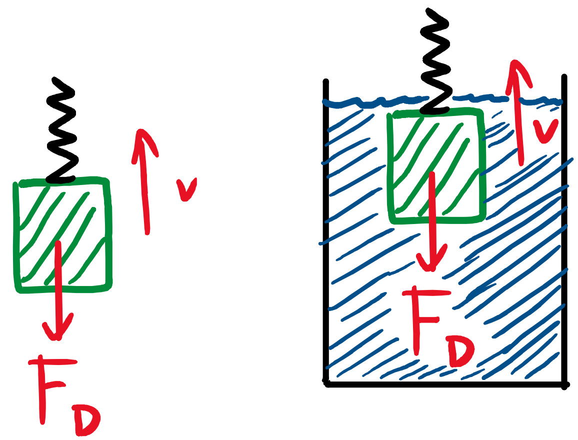
EXTRA: Can you find t_0 exactly given that x(5s) = 4cm?

Forces that lead to damping are velocity dependent & opposite direction to velocity.

examples:

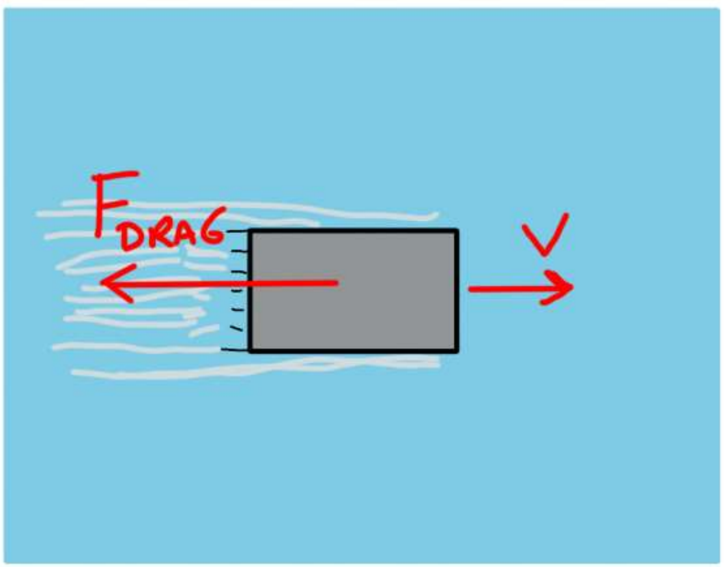


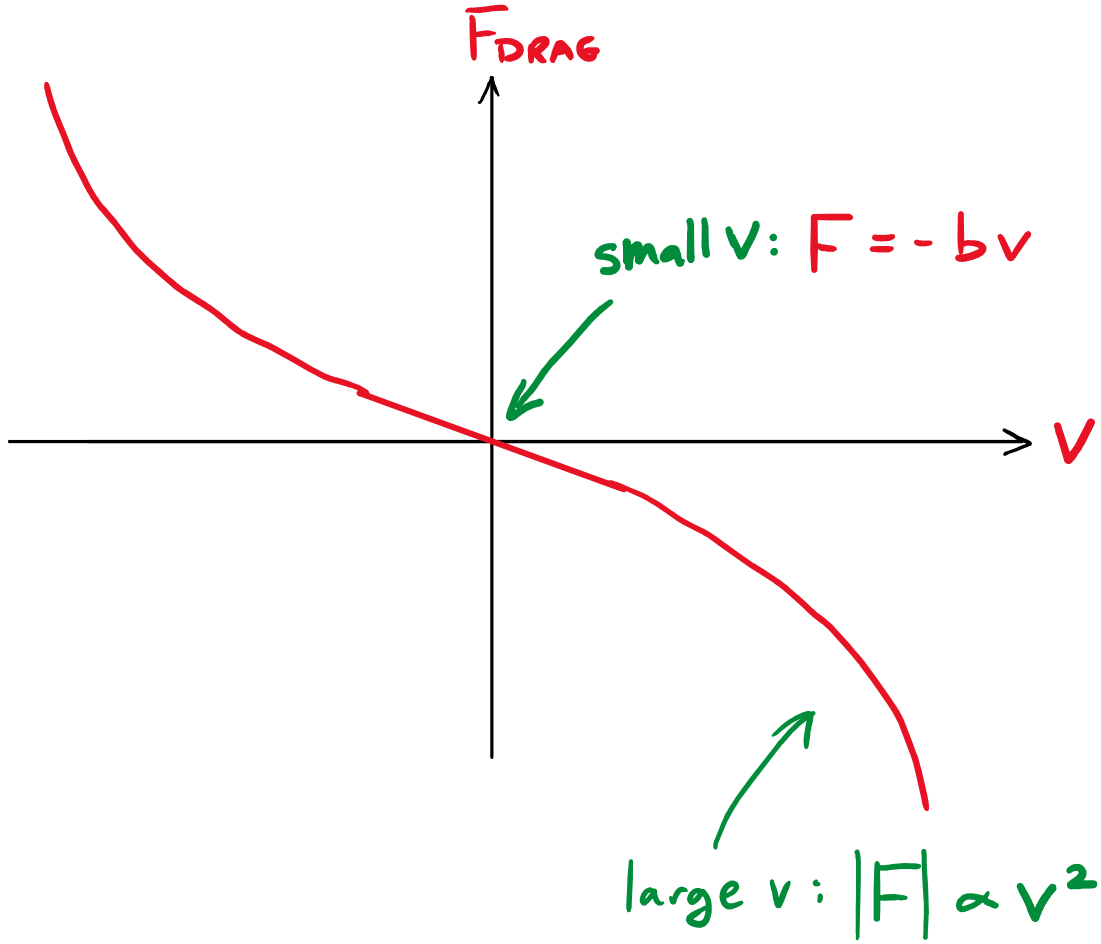
friction



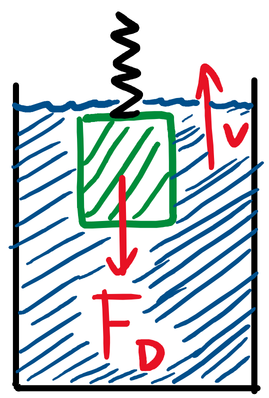
drag forces in air or fluids

Example: drag forces from air/fluids





Example: viscous fluid drag



damping constant

$$F_D = -bv$$

$$F_{NET} = -kx - bv$$

Equations of motion:

$$\frac{dx}{dt} = v$$

This is $a = \frac{F}{m}$

$$\frac{dv}{dt} = -\frac{k}{m}x - \frac{b}{m}v$$

use these to predict how x and v change with time

$$\frac{dx}{dt} = v \qquad \frac{dv}{dt} = -\frac{k}{m}x - \frac{b}{m}v$$

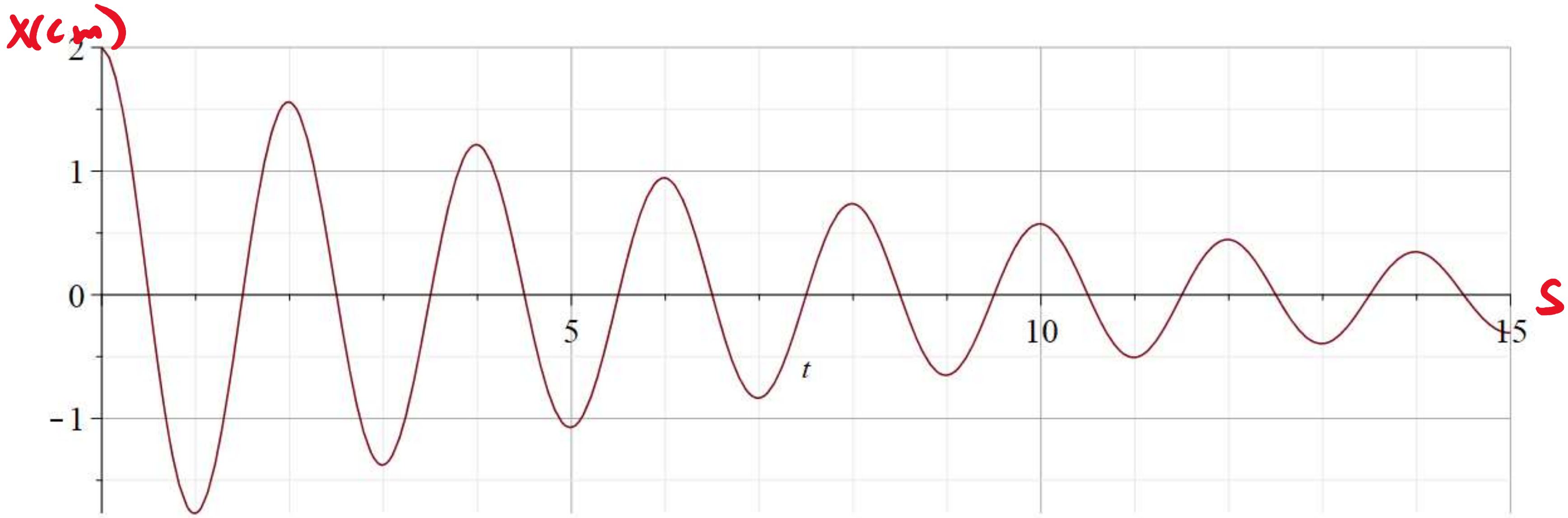
Solution is:

$$x(t) = A_0 e^{-\frac{t}{t_0}} \cos(\omega t + \phi)$$

check: calculate $v = \frac{dx}{dt}$ and then verify 2nd eqn.

$$t_0 = \frac{2m}{b} \qquad \omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

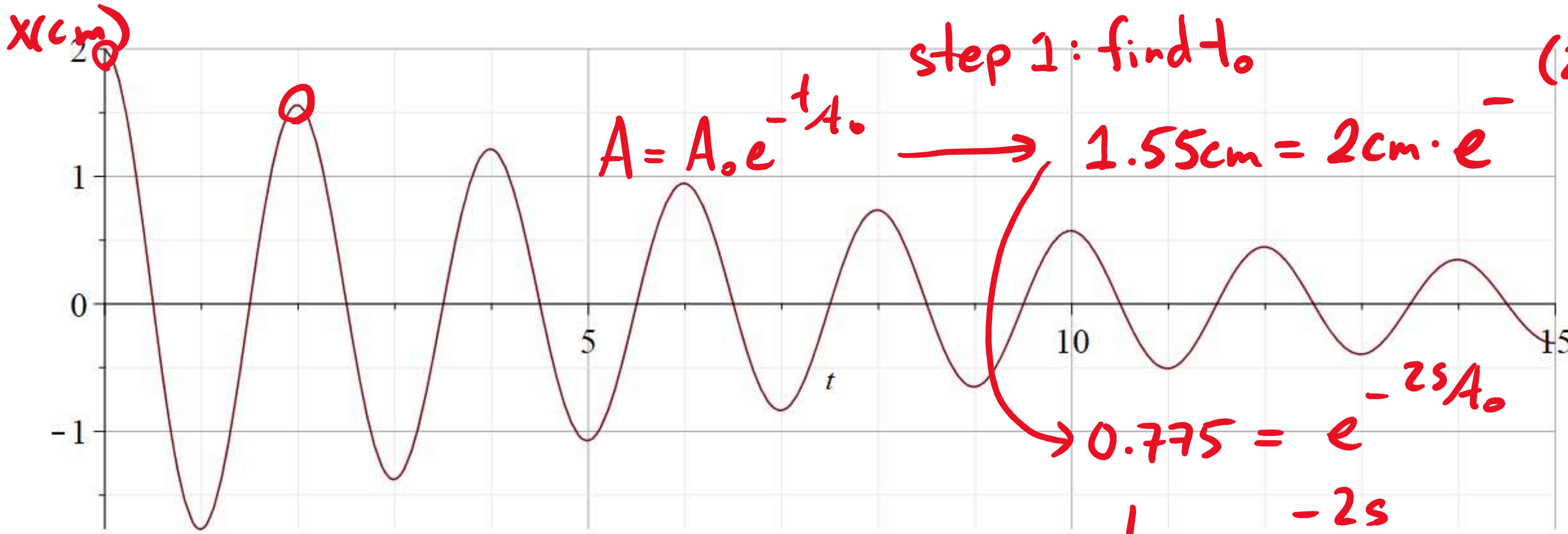
valid for $b < 2\sqrt{km}$



An object with mass 2kg oscillates on a spring with a small amount of damping.

a) What is the damping constant b?

EXTRA: What is the spring constant k?



step 1: find t_0

$A = A_0 e^{-t/t_0}$

$1.55\text{cm} = 2\text{cm} \cdot e^{-(2s)/t_0}$

$0.775 = e^{-2s/t_0}$

$t_0 = \frac{-2s}{\ln(0.775)} = 7.85s$

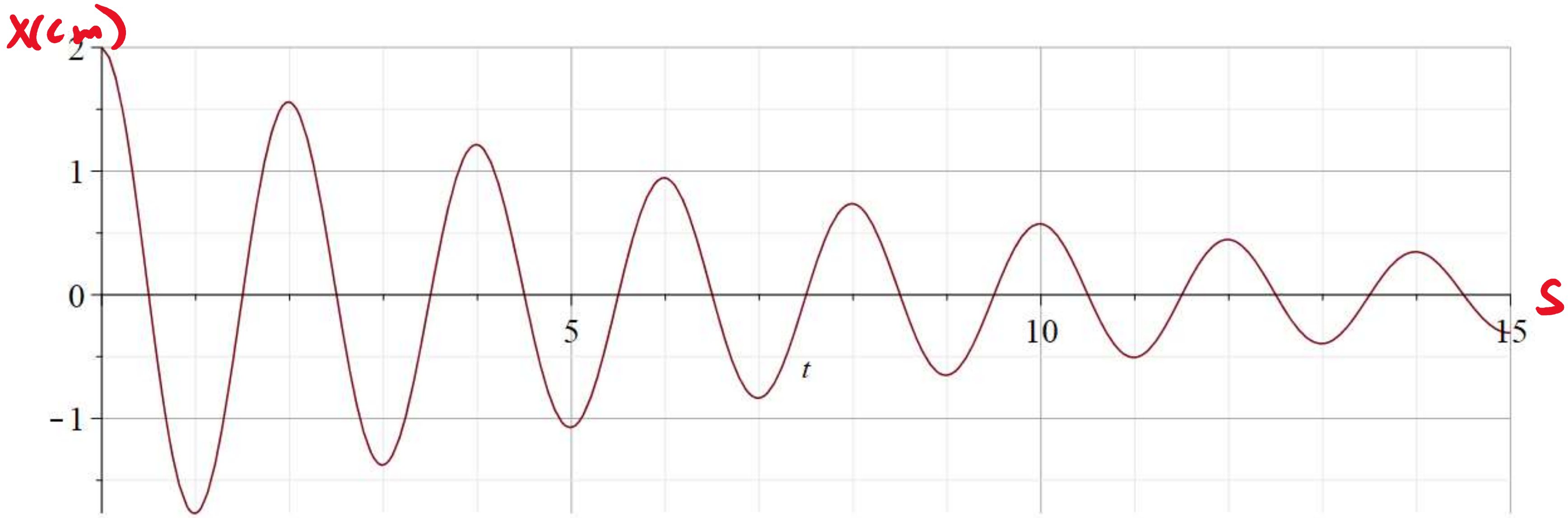
An object with mass 2kg oscillates on a spring with a small amount of damping.

a) What is the damping constant b?

step 2: find b:

$t_0 = \frac{2m}{b} \Rightarrow b = \frac{2m}{t_0} = \frac{4kg}{7.85s} = 0.51 \frac{kg}{s}$

EXTRA: What is the spring constant k?



An object with mass 2kg oscillates on a spring with a small amount of damping.

a) What is the damping constant b?

EXTRA: What is the spring constant k?

step 1: find ω $T = 2s$ so $\omega = \frac{2\pi}{T} = 3.1 s^{-1}$

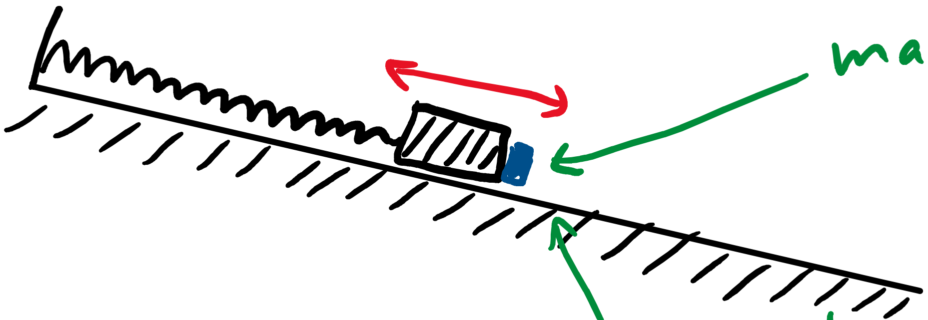
step 2: use $\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

$\Rightarrow k = m\omega^2 + \frac{b^2}{4m}$

$= 19.2 \frac{N}{m} + 0.03 \frac{N}{m} \approx 19.2 \frac{N}{m}$

$\star$ accurate to just use $\omega = \sqrt{\frac{k}{m}}$ unless highly damped.

DEMO



non-magnetic conducting surface

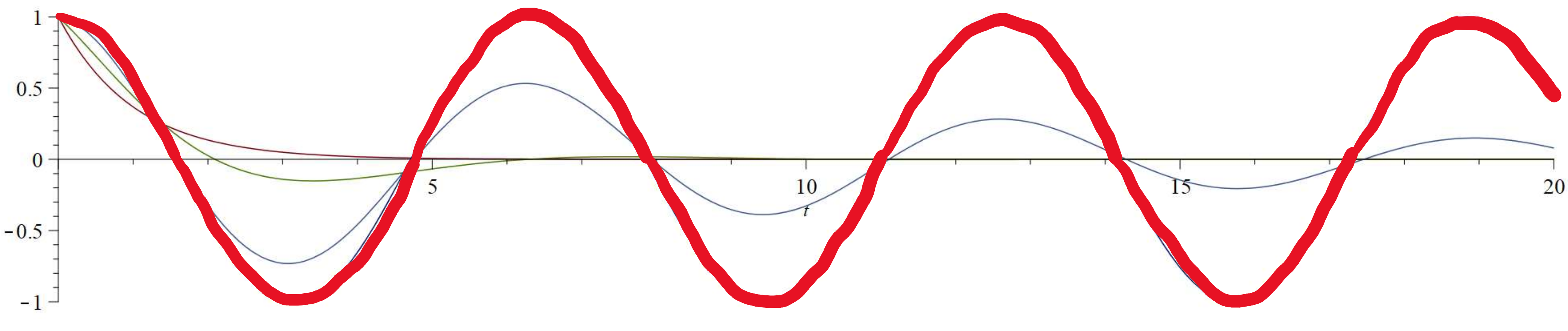
magnet : provides damping

$$F_D = -bv$$

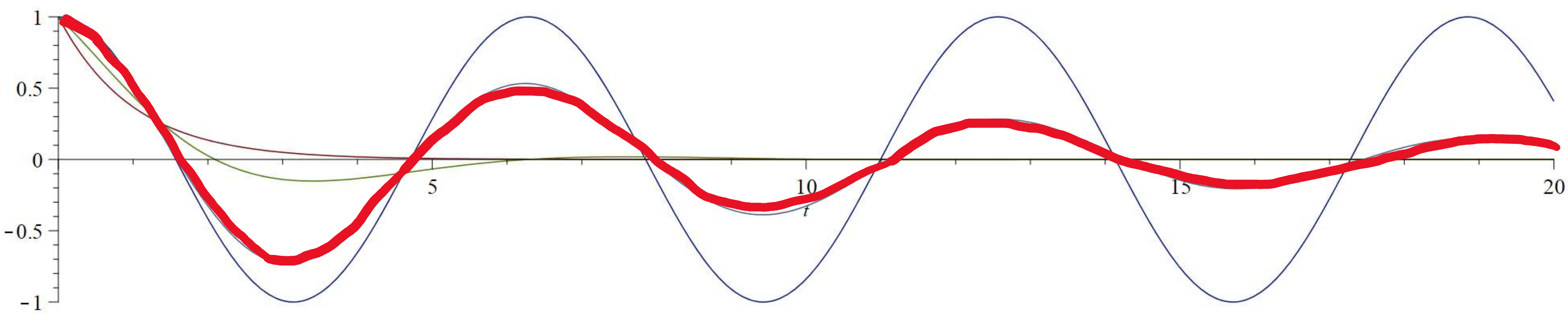
with b controlled by height of magnet

* you can read more about this by searching up "magnetic braking *

no damping (idealized situation)

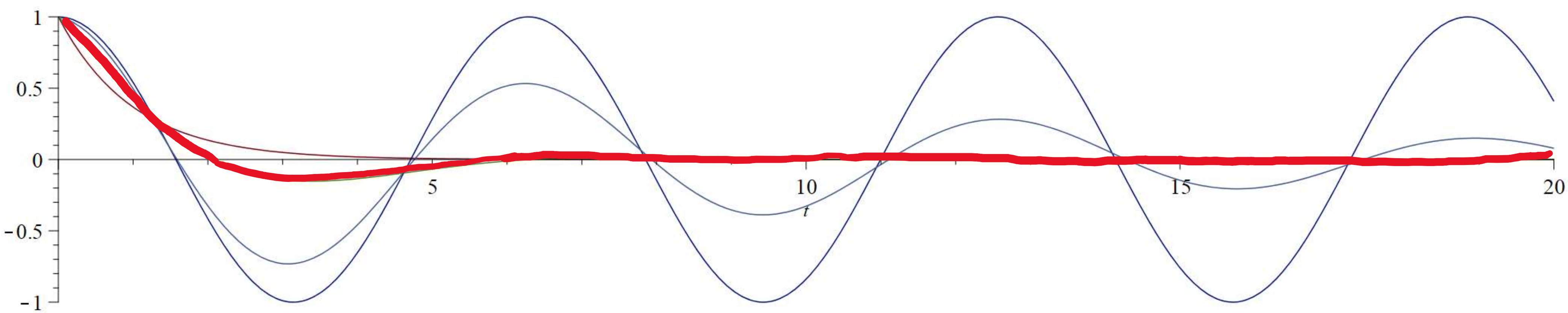


b=0



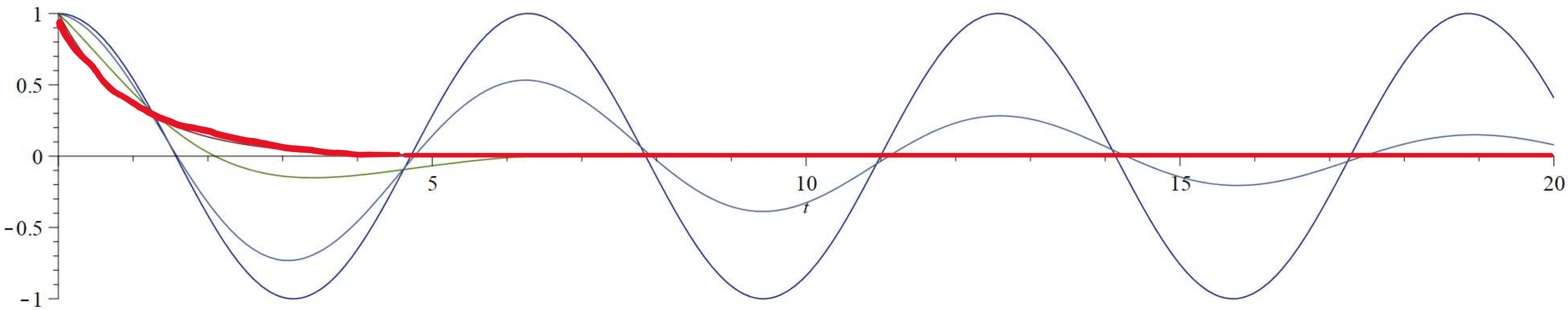
$b = 0.1 \times 2\sqrt{km}$

still have $\omega \approx \omega_{b=0} = \sqrt{\frac{k}{m}}$



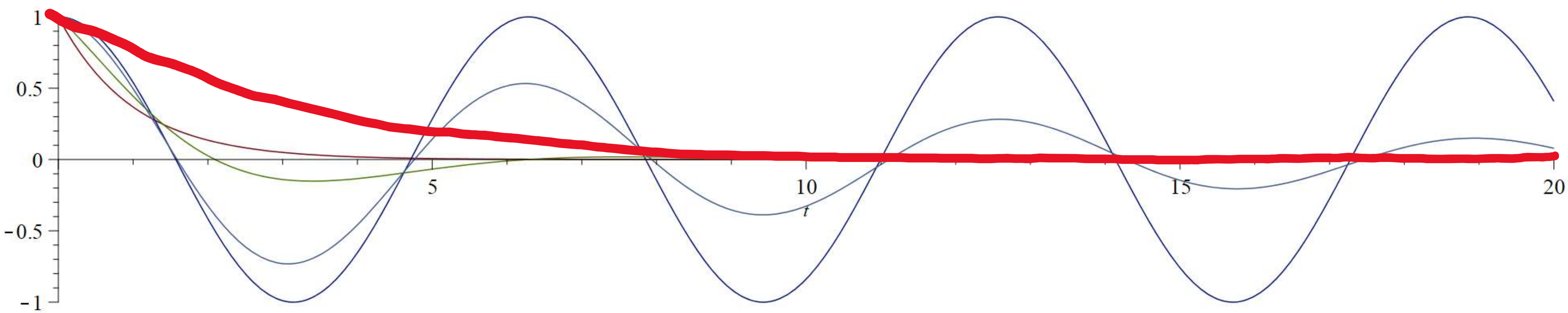
$b = 0.5 \times 2\sqrt{km}$

Critical damping



$b = 2\sqrt{km} \Rightarrow \omega = 0$ pure decay, no oscillations

Overdamping: $b > 2\sqrt{km}$



also exponential decay, but slower to reach equilibrium than critical damping