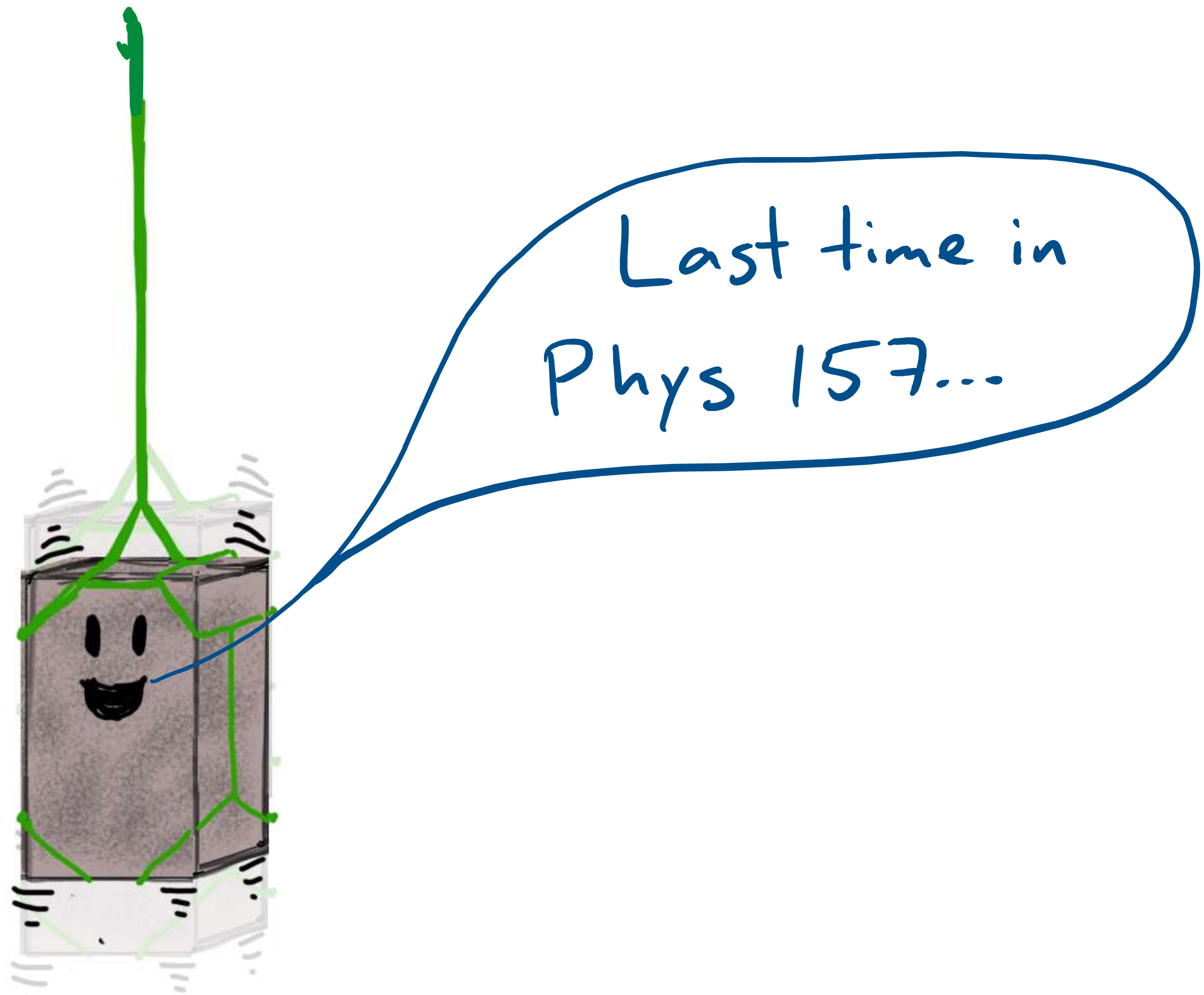
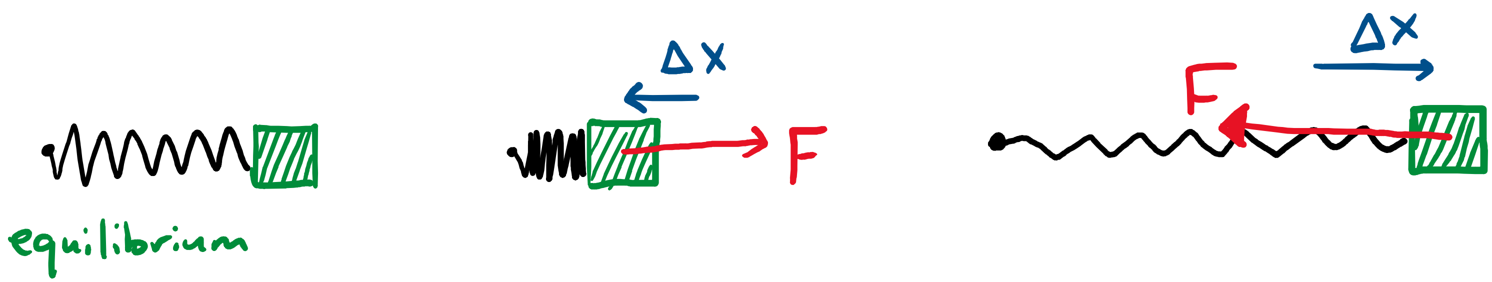

Last time in Phys 157...

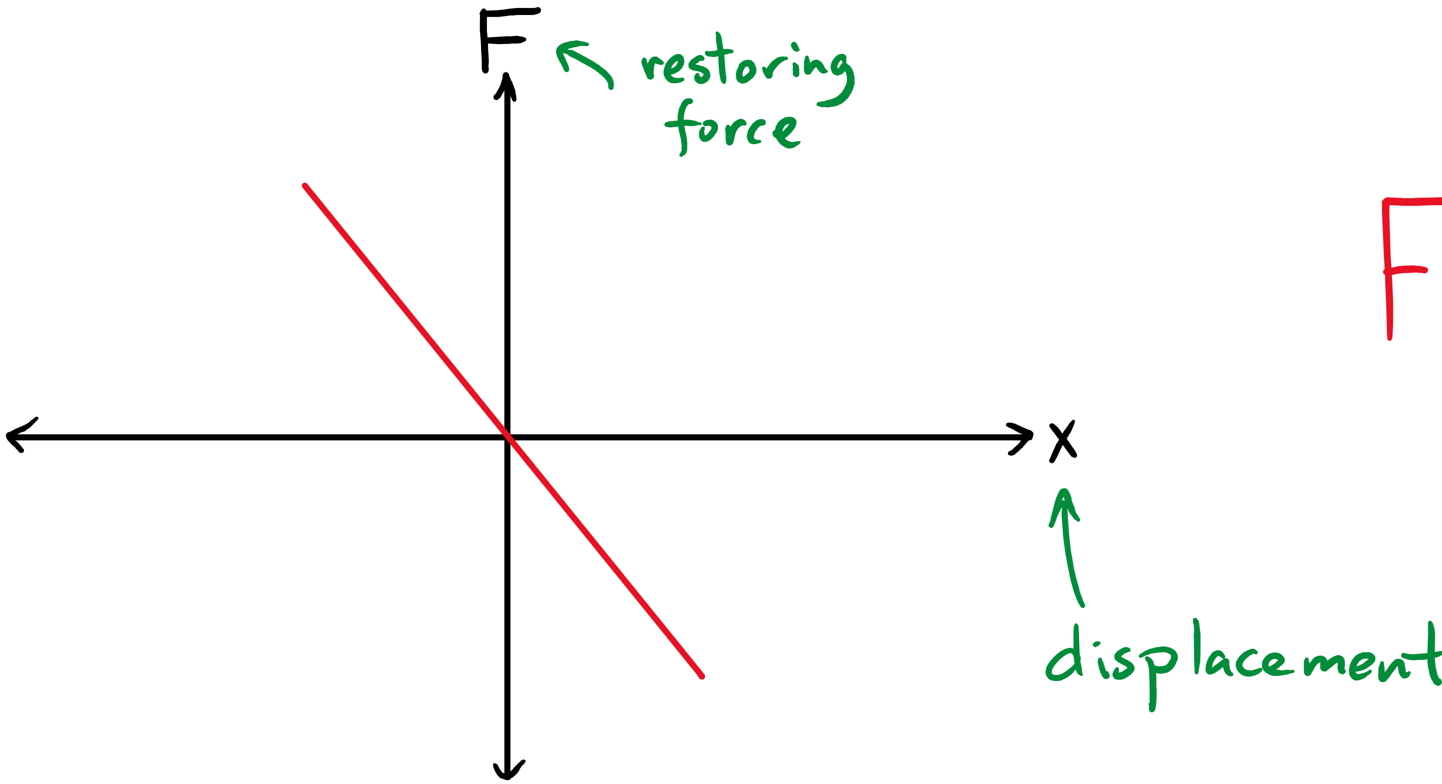
RESTORING FORCES: For an object in STABLE equilibrium, a displacement in one direction leads to a net force in the other direction.

e.g.



This leads to oscillations.

Hooke's Law: Applies to almost any system perturbed a small amount from stable equilibrium

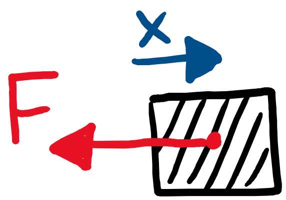


$$F = -kx$$

exact for "ideal spring"

Oscillations with Hooke's Law:

Newton: $a = \frac{F}{m} = -\frac{k}{m} \cdot x$



$F = -kx$

$$\frac{dv}{dt} = -\frac{k}{m} x$$

$$\frac{dx}{dt} = v$$

We can predict how velocity and position change with time.

Solution is $x(t) = A\cos(\omega t + \phi)$ with $\omega = \sqrt{\frac{k}{m}}$

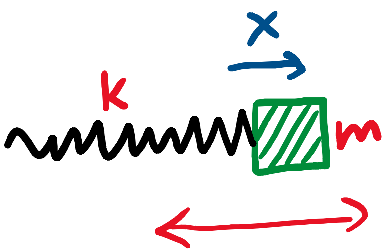
Simple Harmonic Motion

$$x(t) = A\cos(\omega t + \phi)$$

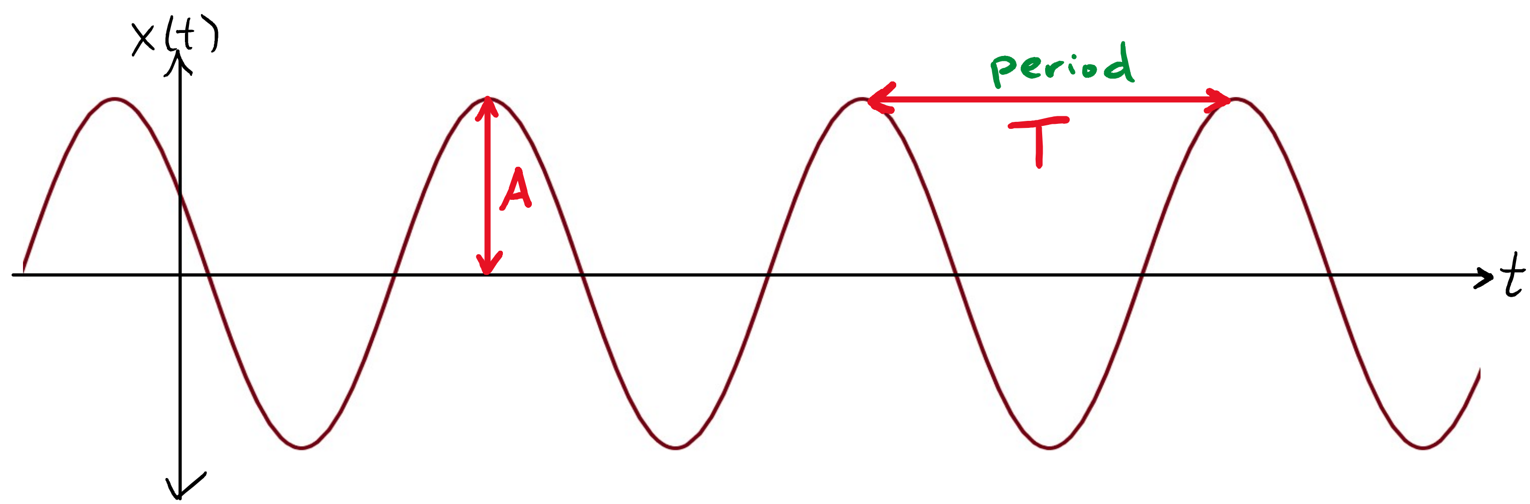
Amplitude

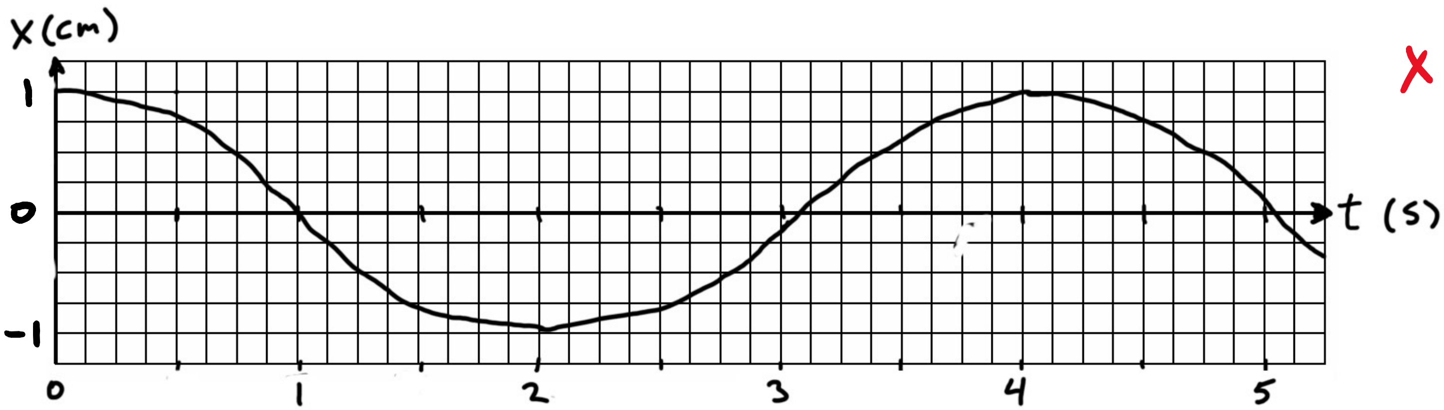
angular frequency

phase



$$\omega = \sqrt{\frac{k}{m}}$$



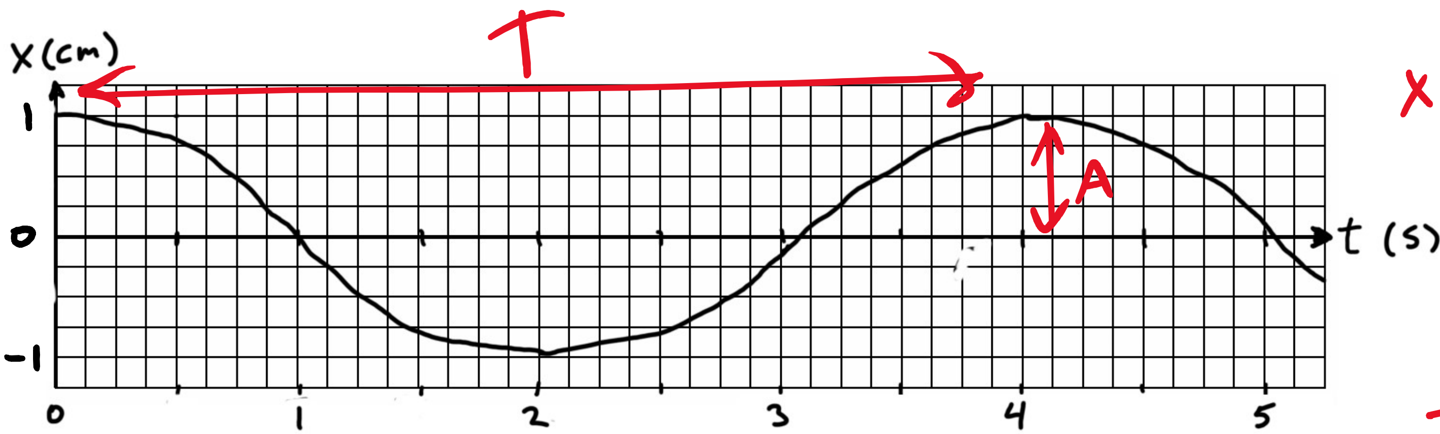


$x(t) = A\cos(\omega t + \phi)$

A plot of ***displacement*** (in cm) as a function of time (in s) is shown above. What are the ***period*** and ***amplitude*** of this simple harmonic motion?

A) T = 1s, A = 2cm
B) T = 2s, A = 2cm
C) T = 4s, A = 2cm
D) T = 2s, A = 1cm
E) T = 4s, A = 1cm

EXTRA: what is ω?



$x(t) = A\cos(\omega t + \phi)$

$A = 1\,cm$

$T = 4s$

A plot of ***displacement*** (in cm) as a function of time (in s) is shown above. What are the ***period*** and ***amplitude*** of this simple harmonic motion?

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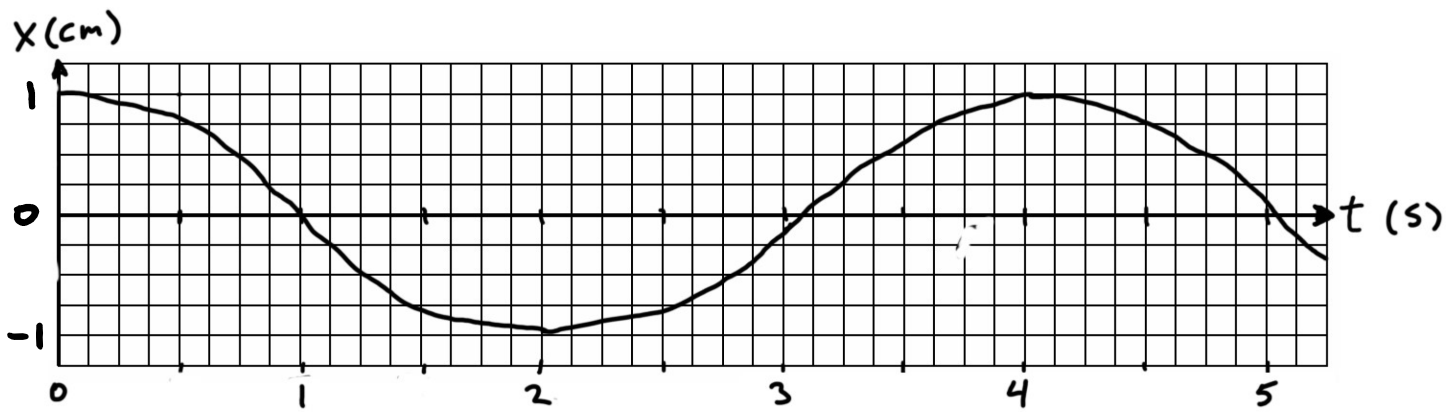


A plot of ***displacement*** (in cm) as a function of time (in s) is shown above. Which function below describes this motion?

A) x(t) = cos(t)
B) x(t) = cos(4t)
C) x(t) = cos(2 π t)
D) x(t) = cos(π t)
E) x(t) = cos(π/2 t)

period of cos is 2π

graph is $\cos(\omega t)$: when $t = 4s$, graph goes back to 1, so must have $\omega t = 2\pi$ here.

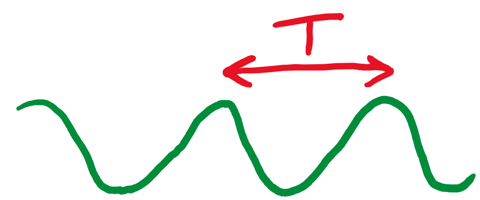
$$\omega = \frac{2\pi}{4s} = \frac{\pi}{2} \ s^{-1}$$

Frequency & Period

angular frequency

$$x(t) = A\cos(\omega t + \phi)$$

Period T : time from max → max

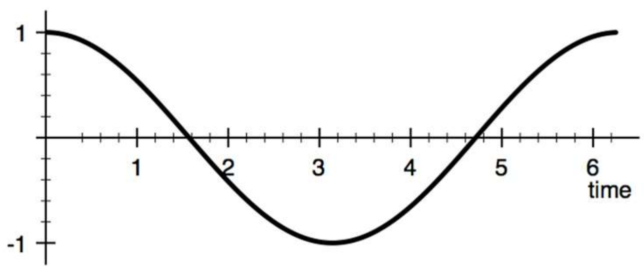


$T = \frac{2\pi}{\omega}$ since cos repeats every 2π.

Frequency f : oscillations per time $f = \frac{1}{T}$

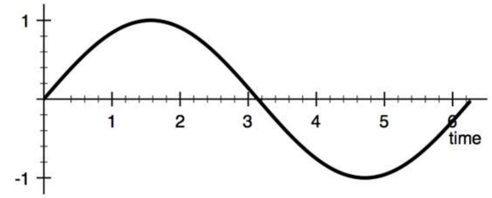
gives: $\omega = 2\pi f$

Simple Harmonic Motion

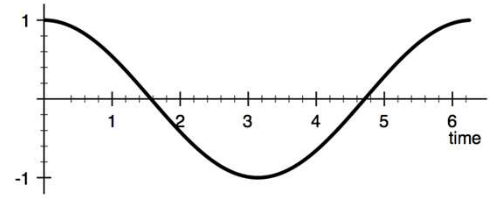


A plot of displacement as a function of time is shown above. Which of the diagrams to the right describes the ***velocity*** as a function of time for the same motion?

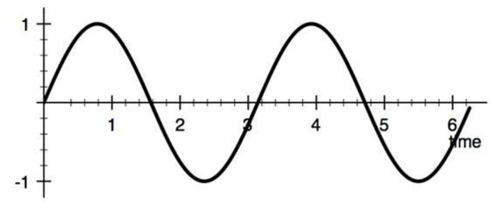
A.



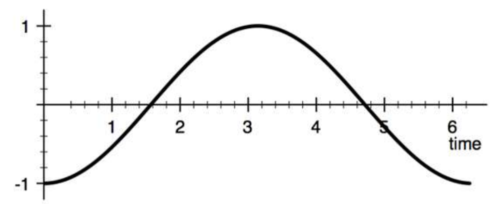
B.



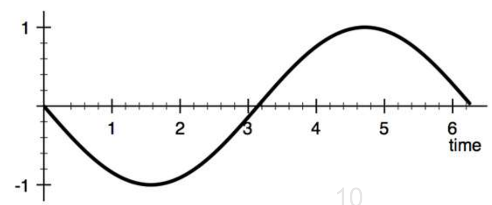
C.



D.



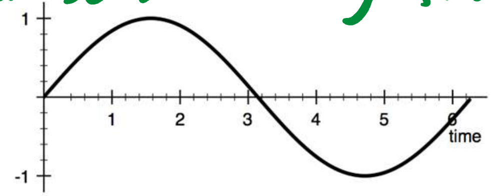
E.



Simple Harmonic Motion

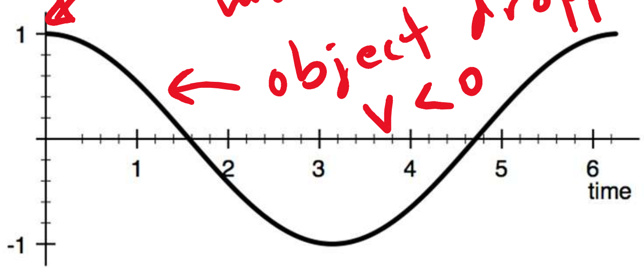
because switching from $v>0$ to $v<0$

A.

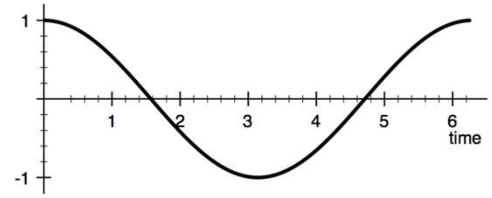


object at top of motion here, so $v=0$

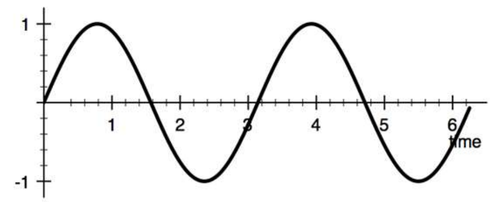
object dropping, so $v<0$



B.

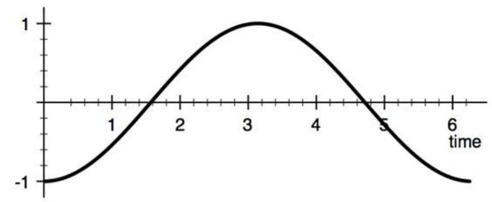


C.



A plot of displacement as a function of time is shown above. Which of the diagrams to the right describes the ***velocity*** as a function of time for the same motion?

D.

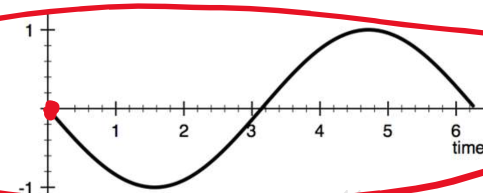


other method:

$$v = \frac{dx}{dt}$$

= slope of graph

E.

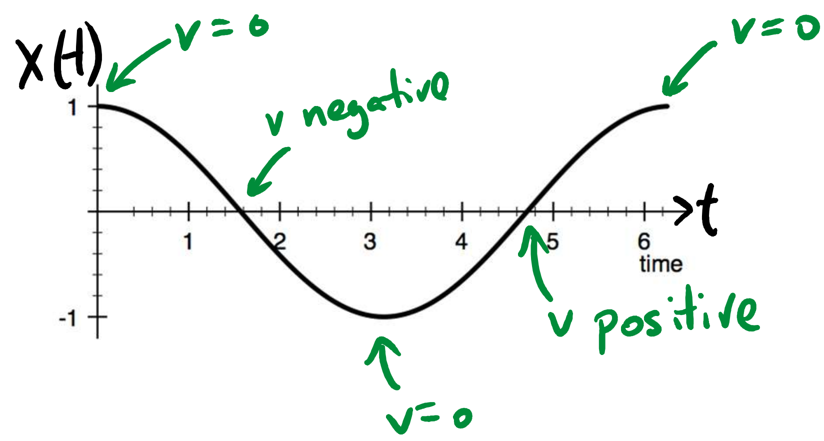


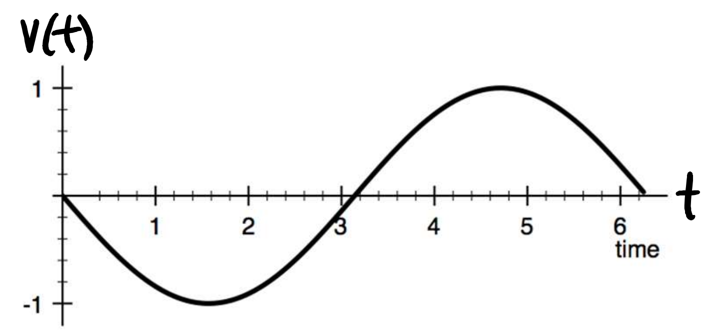
v starts at 0 and goes negatie

Velocity from displacement:

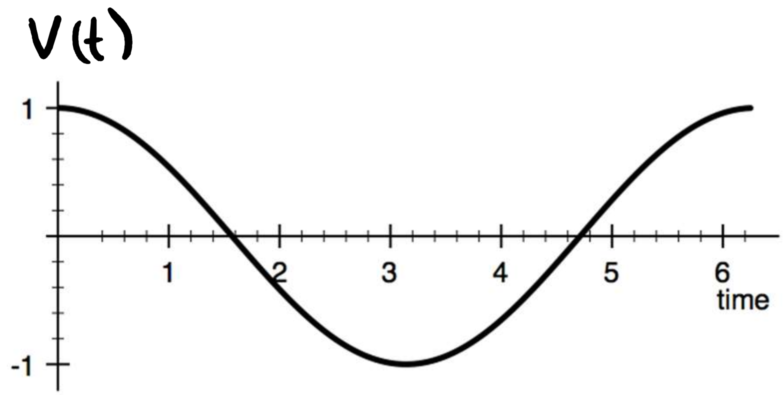
$$v = \frac{dx}{dt}$$

$v(t)$ = slope of $x(t)$
at time t

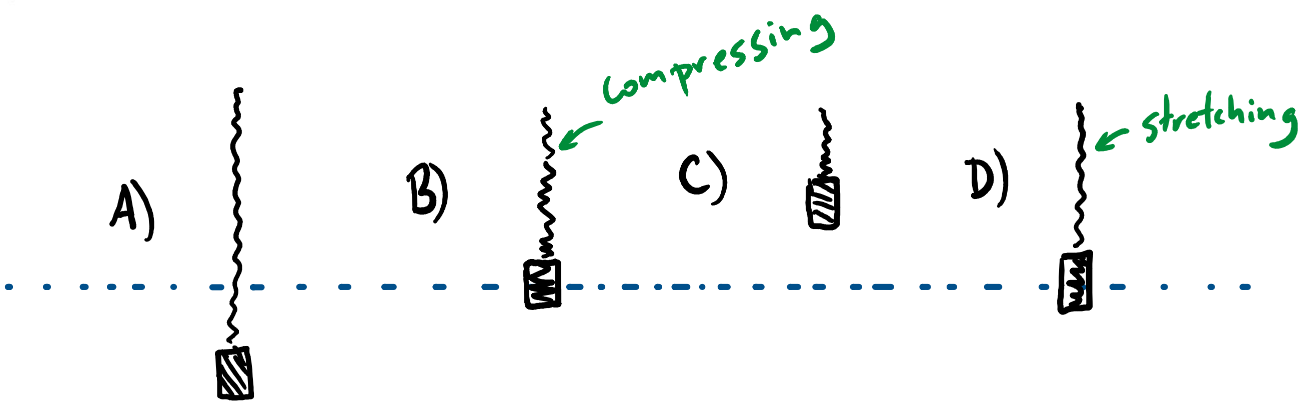




Simple Harmonic Motion:

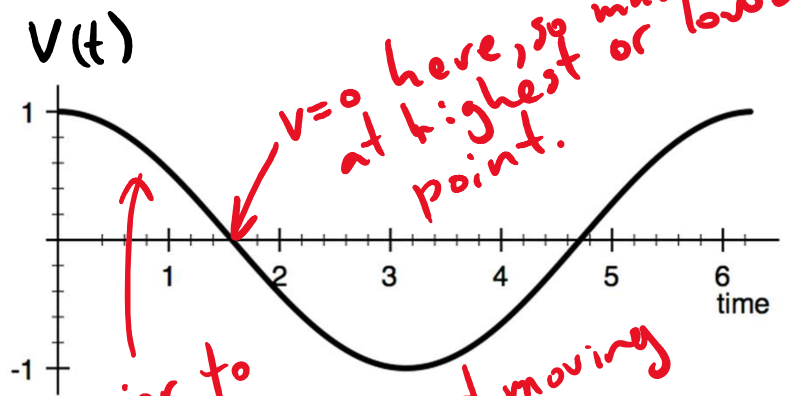


A plot of upward ***velocity*** (in cm/s) as a function of time (in s) is shown above for a mass hanging from a spring. Which of the pictures best represents the situation at t=1.6s?



EXTRA: what does x(t) look like?

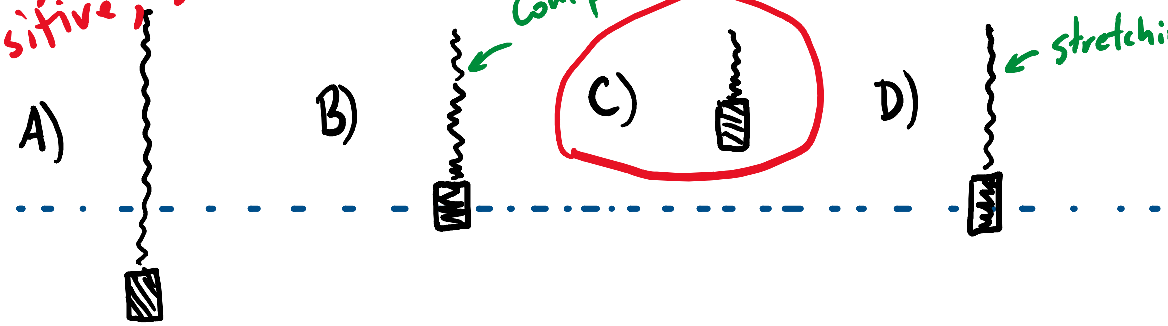
Simple Harmonic Motion:



v=0 here, so must be at highest or lowest point.

prior to t=1.6, v was positive, so object moving up

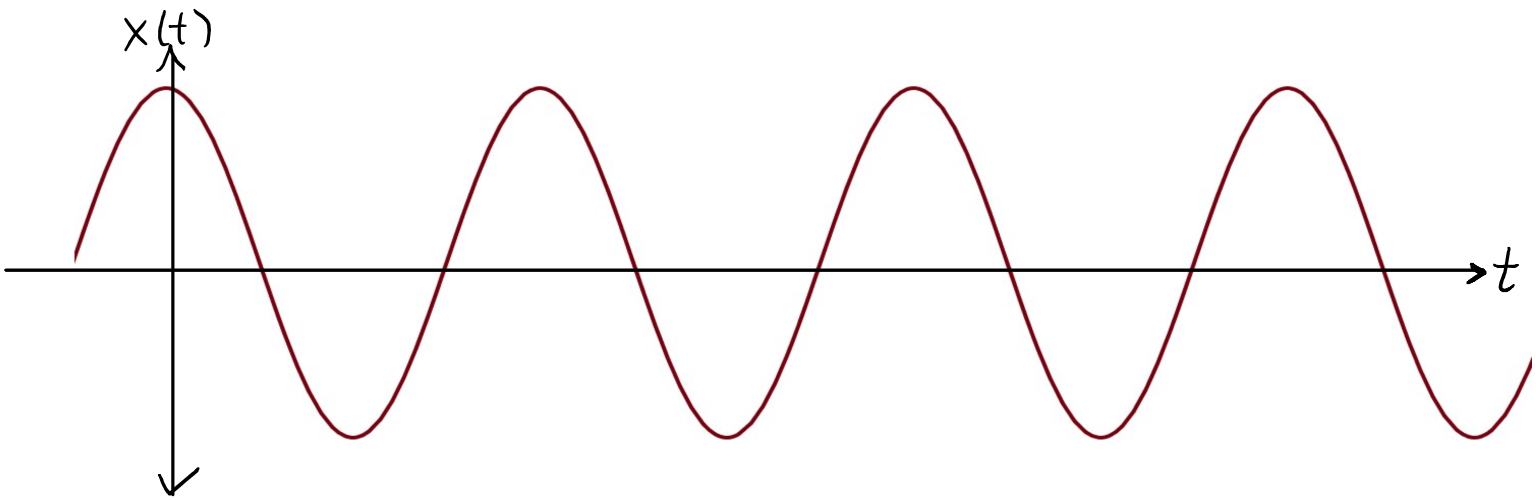
A plot of upward ***velocity*** (in cm/s) as a function of time (in s) is shown above for a mass hanging from a spring. Which of the pictures best represents the situation at t=1.6s?



compressing

stretching

EXTRA: what does x(t) look like?



The graph shows a displacement $x(t) = A\cos(\omega t)$. Adding a small positive phase $x(t) = A\cos(\omega t + \phi)$ will

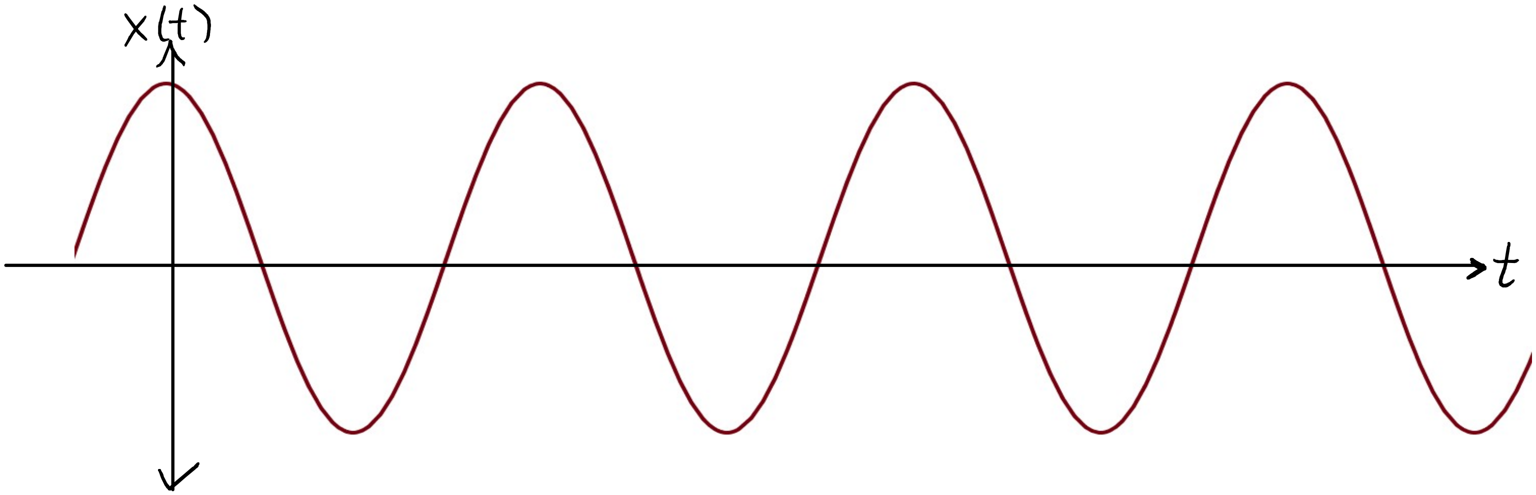
A) Shift the graph to the right

B) Shift the graph to the left

C) Squish the graph so the peaks are closer together

D) Stretch the graph so the peaks are further apart

E) Both A and C

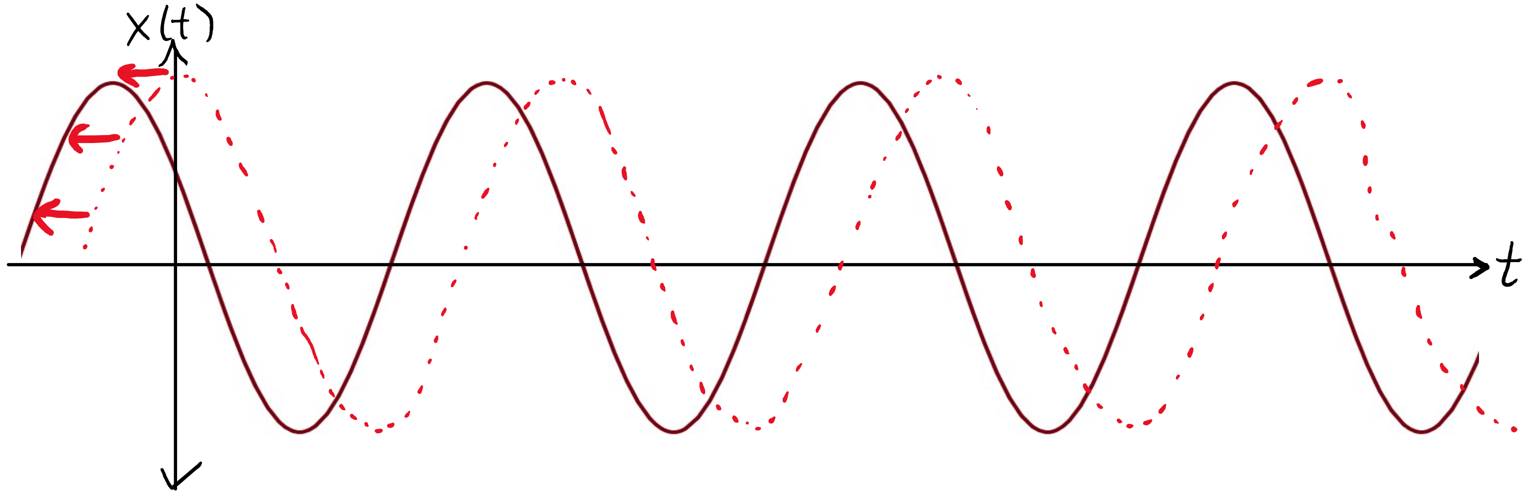


With positive ϕ, we are seeing a later part of the cosine graph than previously, so the graph is shifted to the left

The graph shows a displacement x(t) = Acos(ωt). Adding a small positive phase x(t) = Acos(ωt + ϕ) will

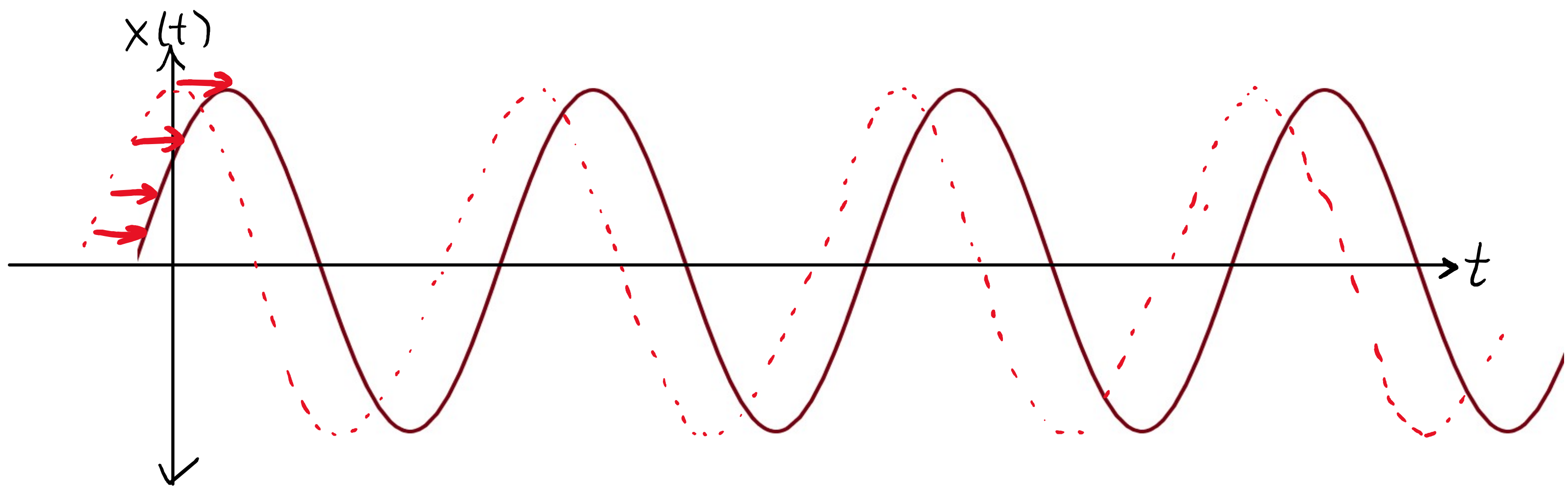
A) Shift the graph to the right
B) Shift the graph to the left
C) Squish the graph so the peaks are closer together
D) Stretch the graph so the peaks are further apart
E) Both A and C

– same as adding $\frac{\phi}{\omega}$ to our time positive ϕ shifts left



$A\cos(\omega t+\phi)$

negative ϕ shifts right



★ shift of 2π is a whole period ★