UBC Physics 157 Conduction Worksheet

In this worksheet, we'll think about how heat transfer proceeds as a function of time based on the conductivity of materials.

Useful formulae:

$H = k A (T_H – T_C)/L$ and $Q = M c \Delta T$

Consider the situation where a block of aluminum at room temperature (T_1 = 20℃) is connected to another equivalent block of aluminum at (T_2 = 220℃) by another strip of aluminum (that has been in place for a while). In this worksheet, assume that the whole system is well-insulated from its environment.

1) On the axes below, sketch graphs (one for each block) showing how you expect the temperatures of the two blocks to behave as a function of time.

2) Now, focusing on just the cooler block, zoom in on the early time part and sketch a graph of how the temperature behaves as a function of time (remembering that the two blocks have already been connected for a while).

3) The original strip was made of aluminum 7 cm long, with 1.6 cm^2 cross section. On the other three graphs below, sketch how your graph from part 2 would look if we instead used:

a) 2 strips of aluminum 7 cm long, each with 1.6 cm^2 cross section

b) a strip of aluminum 14 cm long, with 1.6 cm^2 cross section.

c) a strip of steel 7 cm long, with 1.6 cm^2 cross section.

Note: Aluminum is about 4 times better heat conductor than steel.

a) b) c)

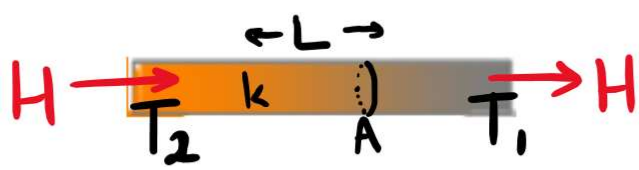
Now let's understand more quantitatively how the graphs relate to the properties of the materials used. We'd like to work out the temperature change ΔT that happens in a small time Δt. Since there are several parts to the system here, it's a good idea to think about each part separately:

4) The cooler block is shown at the left. How much does the block heat up in a time Δt? Answer in terms of H, Δt, M, and c.

Hint: What would the answer be if H = 100 Joules per second and Δt is 0.5 seconds*?*

ΔT =

5) Now, we can think about the strip. Write an expression for the heat current H in terms of the quantities shown.



H =

6) Combining your results for the last two questions, write an expression predicting the slope of the graph in part 2 in terms of T_1, T_2, k, L, A, c, and M,

ΔT/ Δt =

7) Looking back at your graph from part 1, can you explain why the slope changes with time based on your formula from part 5?

7) EXTRA: How would your graph in part 2 change if the strip of aluminum connecting the blocks was originally all at room temperature? Sketch the what you expect and explain.

8) SUPER BONUS: Let’s define $D(t) = T_2(t) - T_1(t)$ to be the temperature difference as a function of time. Using your result from part 6 in the limit where Δt is infinitesimal, can you write down an expression for the derivative of the function $D(t)$, in terms of the function $D(t)$ itself and the constants k, M, c, L, and A? Can you use your result to deduce what kind of function $D(t)$ is?