HOMEWORK 2 SOLUTIONS

1a) For a constant volume gas thermometer, temperature is proportional to pressure, so:

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} = \frac{11\,kPa}{10\,kPa} = 1.1$$

We have $T_1 = T_{triple} = 273.16\,K$, so:

$$T_2 = 1.1 \times T_1 = 1.1 \times 273.16\,K = 300.48\,K$$

b) The volume of the container increases as the steel expands. The change is:

$$\Delta V = \beta \cdot V_0 \cdot \Delta T \qquad \text{where } \beta = 3\alpha = 3.6 \times 10^{-5} K^{-1}$$

The percent change is

$$100 \times \frac{\Delta V}{V_0} = 100 \times \beta \times \Delta T$$

$$= 100 \times 3.6 \times 10^{-5} K^{-1} \times 17.32\,K$$

$$\approx 0.06\,\%$$

2. We want to know the change in temperature so that the iron ring diameter will be the same as the bronze finger diameter. We need:

$$\Delta L_{ring} = \Delta L_{finger} + 0.001\,cm \qquad (1)$$

We have: $\Delta L_{ring} = \alpha_{iron} (L_0)_{ring} \cdot \Delta T \qquad (2)$

$\Delta L_{finger} = \alpha_{bronze} (L_0)_{finger} \cdot \Delta T \qquad (3)$

Substituting ② and ③ into ①, we get:

$$\alpha_{iron}(L_o)_{ring}\,\Delta T = \alpha_{bronze}(L_o)_{finger}\,\Delta T + 10^{-5}m$$

Isolating ΔT:

$$\left[\alpha_{iron}(L_o)_{ring} - \alpha_{bronze}(L_o)_{finger}\right]\Delta T = 10^{-5}m$$

So:
$$\Delta T = \frac{10^{-5}m}{\alpha_{iron}(L_o)_{ring} - \alpha_{bronze}\cdot(L_o)_{finger}}$$

$$= \frac{10^{-5}m}{(1.2\times10^{-5}K^{-1}\cdot 0.02m \; - 1.8\times10^{-5}K^{-1}\times 0.02001m)}$$

$$= -83.2\,K$$

So the ring will fit on below $-63.2\,°C$.

Part b): The net change in circumference of the ring is determined by the change in circumference of the bronze finger, given by

$$\Delta L = \alpha_{bronze}\cdot L_o\cdot\Delta T \qquad \Delta T = +83.2°C$$

For the ring, we know that the change in length is equal to the change due to thermal expansion plus the change due to stress:

$$\Delta L = \alpha_{iron}\cdot L_o\cdot\Delta T + \frac{F}{A}\cdot\frac{L_o}{Y_{iron}}$$

Setting these two expressions for ΔT equal to each other, we have:

$$\alpha_{bronze} \cdot L_0 \cdot \Delta T = \alpha_{iron} \cdot L_0 \cdot \Delta T + \frac{F}{A} \cdot \frac{L_0}{Y_{iron}}$$

Solving for the stress $\frac{F}{A}$, we find the stress:

$$\frac{F}{A} = \frac{Y_{iron}}{L_0}\left(\alpha_{bronze} \cdot L_0 \cdot \Delta T - \alpha_{iron} \cdot L_0 \cdot \Delta T\right)$$

$$= Y_{iron}\left(\alpha_{bronze} - \alpha_{iron}\right) \cdot \Delta T$$

$$= (21 \times 10^{10} \quad 0.6 \times 10^{-5} \cdot 83.2)\ Pa$$

$$= 1.05 \times 10^{8}\ Pa$$

3. Between 0°C and 30°C, both the gas and the tank expand. We have:

$$\Delta V_{gas} = \beta_{gasoline} \cdot V_0 \cdot \Delta T$$

$$= 9.5 \times 10^{-4} K^{-1} \cdot 50L \cdot 30K$$

$$= 1.43L$$

$$\Delta V_{tank} = \beta_{alum.} \cdot V_0 \cdot \Delta T \quad \text{with } \beta_{al} = 3\alpha_{alum.}$$

$$= 3 \times (2.4 \times 10^{-5} K^{-1}) \times 50L \cdot 30K$$

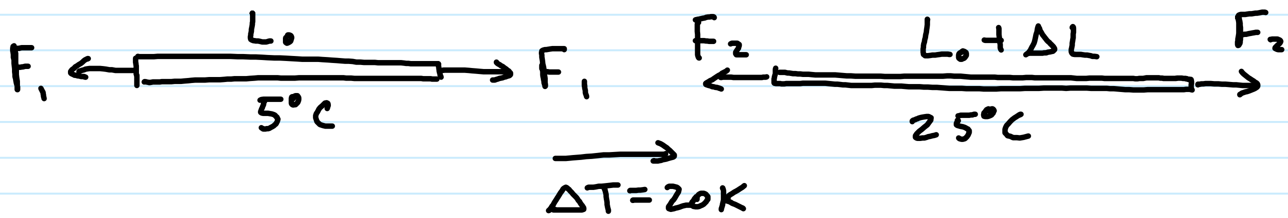
$$= 0.11L$$

So the excess volume of gasoline is 1.32L, and the tube needs to have at least this volume.

4. The net expansion of the wire is fixed by how much the gold expands. We have:

$$\Delta L_{gold} = \alpha_{gold} \cdot L_0 \cdot \Delta T$$

for the distance between the points where the wire is fixed.



The wire also changes length by this amount, but this is due to the combined effects of thermal expansion and a change in tension. We have:

$$\Delta L_{wire} = \alpha_{plat.} \cdot L_0 \cdot \Delta T + \frac{\Delta F}{A} \cdot \frac{L_0}{Y_{plat.}}$$

Since $\Delta L_{wire} = \Delta L_{gold}$, we get:

$$\alpha_{gold} \cdot L_0 \cdot \Delta T = \alpha_{plat} \cdot L_0 \cdot \Delta T + \frac{\Delta F}{A} \cdot \frac{L_0}{Y_{plat.}}$$

$$\Rightarrow \quad \frac{\Delta F}{A} \cdot \frac{L_0}{Y_{plat}} = (\alpha_{gold} - \alpha_{plat.})\, L_0\, \Delta T$$

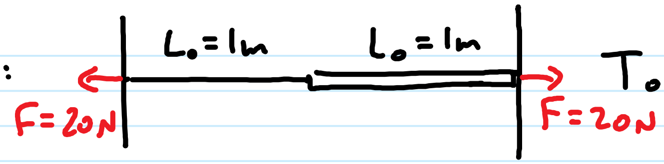
$$\Rightarrow \quad \Delta F = A \cdot Y_{plat} \cdot (\alpha_{gold} - \alpha_{plat})\, \Delta T$$

$$= \pi \cdot (0.5 \times 10^{-3})^2 \cdot 16 \times 10^{10} \times (0.5 \times 10^{-5}) \cdot 20 \text{ N}$$

$$= 12.6 \text{ N}$$

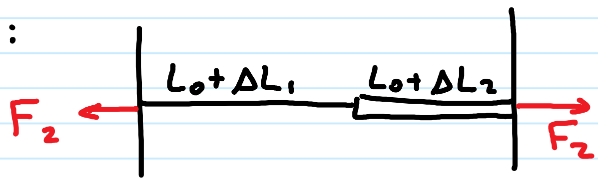
So you should set the tension to 37.4 N.

(5) Before:



$\downarrow \Delta T = -20K$

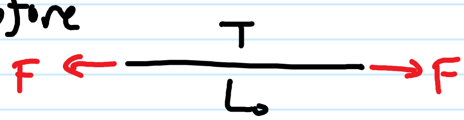
After:



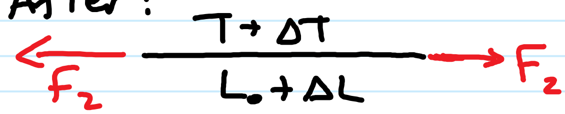
$\Delta L_1 + \Delta L_2 = 0$

For each of the two parts, we have a change in length due to the combined effects of thermal contraction and a change in temperature:

Before



After:



We get:

thermal part

change due to change in tension.

(*) nylon: $$\Delta L_1 = \alpha_1 \cdot L_0 \cdot \Delta T + \frac{\Delta F}{A_1} \cdot \frac{L_0}{Y_1}$$

steel: $$\Delta L_2 = \alpha_2 L_0 \cdot \Delta T + \frac{\Delta F}{A_2} \cdot \frac{L_0}{Y_2}$$

The total length is fixed, so $\Delta L_1 + \Delta L_2 = 0$. This gives:

$$\alpha_1 L_0 \cdot \Delta T + \alpha_2 L_0 \Delta T + \frac{\Delta F}{A_1} \cdot \frac{L_0}{Y_1} + \frac{\Delta F}{A_2} \cdot \frac{L_0}{Y_2} = 0$$

Solving for the unknown quantity ΔF, we get:

$$\Delta F = \frac{-\Delta T \cdot (\alpha_1 + \alpha_2)}{\left(\frac{1}{Y_1 A_1} + \frac{1}{Y_2 A_2}\right)} = 11.5N$$

use $A = \pi \left(\frac{D}{2}\right)^2$

Now we can plug this into the equation (★) to find ΔL. We have:

$$\Delta L_1 = \alpha_1 \cdot L_0 \cdot \Delta T + \frac{\Delta F}{A_1} \cdot \frac{L_0}{Y_1}$$

$$= 1.58 \times 10^{-5} m \qquad \text{(nylon)}$$

So the connection point moves 0.0158 mm, with the nylon getting longer.