Physics 157 Homework 2: due Wed, Sept 26th by 5pm

Hand-in boxes for each tutorial section are upstairs in the Life building. Write your name, student number, and tutorial section on your submission.

Classes this past week have focused on the thermal expansion of materials and the resulting mechanical forces (stress) that can be caused by this expansion. In this week's homework you will get practice applying the quantitative relationships that govern these effects. Specific skills you will practice include:

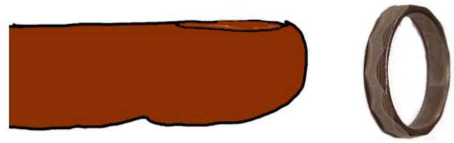
- For an object made of some material, to calculate the changes in length or volume that material undergoes in response to changes in temperature and external forces (stress).
- For systems consisting of two different materials, to quantitatively analyze effects resulting from the different expansion rates of different parts.
- For structures involving multiple parts, to calculate the mechanical forces of various parts of the system on adjacent parts arising from changes in temperature.

For all questions, explain your work. Don't just write equations and give the answer. For examples, see solutions to last week's homework (though point form is okay).

Question 1:

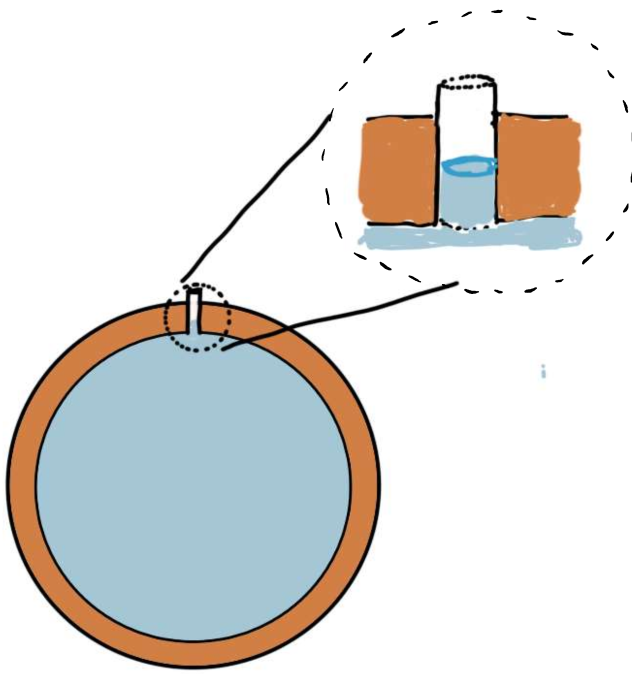
A "constant volume" gas thermometer has a nearly ideal gas in a spherical container with walls of steel ($\alpha = 1.2 \times 10^{-5}\ K^{-1}$). In equilibrium with water at its triple point, the pressure reads 10kPa. The thermometer is now placed in equilibrium with another container of water and the pressure reads 11kPa.

a) What is the temperature of the water in the second container, assuming the thermometer is constant volume?
b) By what percentage has the volume of the container actually increased due to the temperature change?



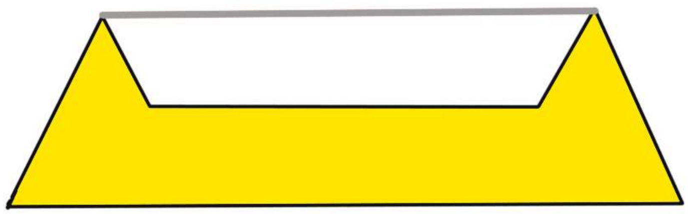
Question 2: You have just completed a larger-than-life bronze statue of an engineer to place next to the engineering cairn on Main Mall, in order to inspire the passing students. To complete the work, you need to place an engineer's iron ring on the finger of the statue. At 20 degrees Celcius, the inner diameter of the iron ring is 2.0000 cm, while the outer diameter of the finger on the statue is 2.0010cm. In order to make the ring fit, you change the temperature of the ring and the finger together until the ring slides on. At what temperature will this be possible? ($\alpha_{iron} = 1.2 \times 10^{-5}\ K^{-1}$, $\alpha_{bronze} = 1.8 \times 10^{-5}\ K^{-1}$)

Extra part (not to be handed in): when the statue returns to 20 degrees Celcius, what will be the stress in the steel ring, assuming the ring doesn't affect the thermal expansion of the statue's finger? (you may want to do question 4 before trying this)



Question 3: You would like to design an aluminum gasoline tank so that at 0 degrees Celcius, the main part of the tank holds 50L of gasoline. There is a small tube at the top of the tank (where gas is added/removed). To avoid spilling due to thermal expansion, you want to make sure this tube is long enough so that the gas can expand into it. What is the minimum volume of this tube if we want to avoid a spill when the tank is filled to the bottom of the tube at 0 degrees Celcius and the temperature rises to 30 degrees Celcius?

($\alpha_{aluminum} = 2.4 \times 10^{-5}\ K^{-1}$, $\beta_{gasoline} = 9.5 \times 10^{-4}\ K^{-1}$)



Question 4: You are the Royal Engineer for the Kingdom of Grrrrrx (pronounced as written). Each year in the kingdom, on the last day of summer, a new Knightship of Grrrrrx is awarded to the winner of the Royal Singing-and-Hopping Race, in which participants (18 years of age and older) must hop and sing through three full laps of the castle perimeter, adhering to the strict regulations of the Royal Singing-and-Hopping Commission.

The race begins when the King of Grrrrrx plucks a single note on the Royal Plucking Instrument, which consists of a single 1mm thick platinum wire stretched between two points on a solid gold frame, as shown in the picture. To achieve the proper note, the wire must be at a tension of 50.00N. On the morning of the race, you notice the temperature is a chilly 5 degrees Celcius but that the temperature is forecast to rise to 25 degrees Celcius by 2pm when the race starts. It is your duty as Royal Engineer to set the tension of the wire, so that it will play just the right note when plucked at 2pm. To what tension should you set it?

($\alpha_{gold} = 1.4 \times 10^{-5}\,K^{-1}$, $\alpha_{platinum} = 0.9 \times 10^{-5}\,K^{-1}$, $Y_{platinum} = 16 \times 10^{10}$ Pa)

Hint: Assume that the expansion of the gold is unaffected by the presence of the wire. You do not need to know the length of the wire.

Question 5 (from last year's midterm):

1. A length of nylon fishing line of 2 mm diameter is tied to a length of 0.5 mm diameter stainless steel wire, then stretched between two rigid supports 2 m apart such that the tension in the wires is 20 N and the connection point between the two wires is exactly midway between the posts. The air temperature then goes down by 20°C. Find the distance that the connection moves.

[Youngs modulus for nylon is 3 GPa, its thermal expansion coefficient is 6×10^{-5}/K. Youngs modulus for stainless steel is 180 GPa, its thermal expansion coefficient is 1.7×10^{-5}/K.]

Hint: This is very similar to the two rods problem from class. The change in tension in the wires is an unknown that you may wish to solve for first.

Old midterm questions on thermal expansion and thermal stress

(for extra practice, not to be handed in)

1. A aluminum rod of density $\rho = 2700\,kg/m^3$ has a length of $3.0\,m$ and cross sectional area $9.0\,cm^2$. It is bolted at both ends between two immobile supports. Initially, there is no tension in the rod because the rod just fits between the supports. Through cooling, the rod loses $3500\,J$ of heat.

a) Find the temperature change from the heat loss.

b) Find the force on the rod that develops from the heat loss.

Useful constants $c_{Al} = 910\,J/(kgK)$, $Y_{Al} = 65 \times 10^9\,Pa$, $\alpha_{Al} = 22 \times 10^{-6} K^{-1}$.

1. The main span of the Lions Gate Bridge has a length of 473 m. On each end there are expansion joints like the one on the photo below. One day, the temperature in Vancouver changed from -4°C to +15°C degrees between 6 AM in the morning and 2 PM in the afternoon.

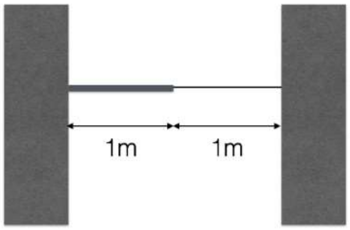
a) What was the *average* speed of a "tooth" in one of the expansion joints?

b) At 6 AM in the morning, a piece of tire rubber fell into one of the cracks, filling it completely. The rubber was 10 cm long and had a cross section of 4 cm^2. What was the stress in the rubber at 2 PM?

Clearly state all of the assumptions that you made while solving this problem.

Data: Young modulus of steel 200 GPa
Young modulus of rubber 7 kPa
Linear Expansion coefficient of steel $13 \times 10^{-6} K^{-1}$
Linear Expansion coefficient of rubber $77 \times 10^{-6} K^{-1}$

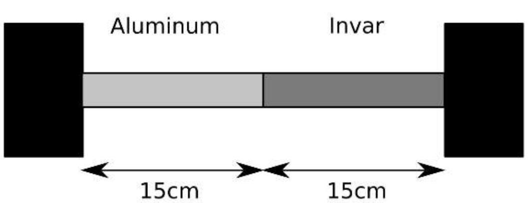
1. Two pillars, separated by 2m, have a cord strung between them that consists of two different wires attached to each other:



The cord consists of a 5mm diameter nylon wire attached to a 1mm diameter copper wire; each segment is 1m long and under no tension at the initial temperature of $25°C$. Then, the cord is tightened to a tension of 100N at $25°C$. Finally, the temperature drops to $5°C$. What is the final tension in the cord? [Youngs modulus for nylon is 3GPa, its thermal expansion coefficient is $1.6 \times 10^{-5}/K$. Youngs modulus for copper is 117GPa, its thermal expansion coefficient is $8 \times 10^{-5}/K$.]

1. An aluminum rod 15 cm long with a diameter of 1.0 cm is fixed between rigid supports with an Invar rod of the same dimensions such that there is no tension or compression of the rod at 20°C. The rods are attached to each other such that they can push or pull without separating. Invar has a negligibly small thermal expansion coefficient. You place this apparatus in an experiment where heat is added or removed and you measure that the Invar rod is a total of 0.25 mm longer than it was at 20°C.

Data: $\alpha_{Al} = 23 \times 10^{-6} K^{-1}$, $Y_{Al} = 69GPa$, $\rho_{Al} = 2.7g/\text{cm}^3$, $c_{Al} = 0.90Jg^{-1}K^{-1}$, $Y_{Invar} = 140GPa$



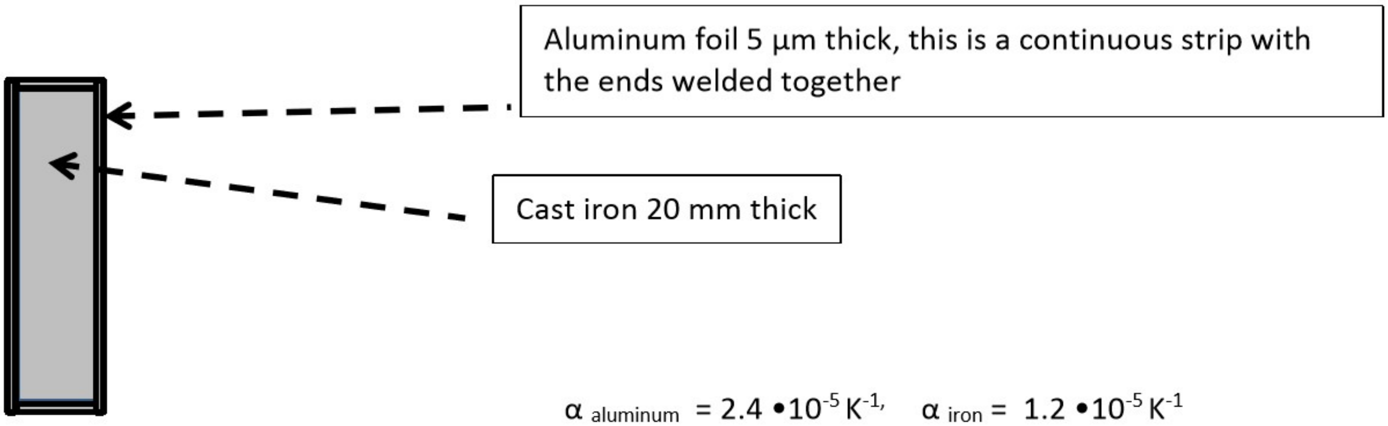
a) Find the final temperature.

b) If the particular Aluminum alloy used has a Yield strength of 200MPa, is the rod likely to be permanently deformed?

c) Was heat added or removed from the Aluminum rod? How much?

Clearly state all of the assumptions that you made while solving this problem.

Problem 2. A block of cast iron was wrapped in the thin aluminum foil as shown on the figure. At room temperature there is no stress on the foil. The whole system is then immersed in liquid nitrogen at -196°C. Find the stress on the aluminum foil. Is the foil going to deform permanently? Is it going to break?



$\alpha_{\text{aluminum}} = 2.4 \bullet 10^{-5}\,K^{-1}$, $\alpha_{\text{iron}} = 1.2 \bullet 10^{-5}\,K^{-1}$

Fig. 2. Stress-strain diagrams for aluminum. Left: Low strain region. Right: Full strain range. Hint: you can get the Young's modulus, proportions limit (elastic limit), and break point from the graphs.