HOMEWORK 1 SOLUTIONS

① A linear relation between gain and temperature means that for any change ΔT in temperature, the change in gain is

$$\Delta G = m \, \Delta T$$

for some constant m. From the data given in the question, $\Delta G = 35.2 - 30.0 = 5.2$ for $\Delta T = 55°C - 20°C = 35C°$, so we must have

$$m = \frac{\Delta G}{\Delta T} = \frac{5.2}{35C°} = 0.15\,(C°)^{-1}$$

The change in gain from 20°C to 30°C is then

$$\Delta G_{20\to 30} = m \cdot (10\,C°) = 1.5$$

So the gain at 30°C is

$$\begin{aligned} G_{30°} &= G_{20°} + \Delta G_{20\to 30} \\ &= 30.0 + 1.5 \\ &= \underline{\underline{31.5}} \end{aligned}$$

ALTERNATE SOLUTION:

A linear relationship means that we have:

$$G = mT + b$$

for some constants m and b. Using the data:

$$30 = m \cdot (20°C) + b \qquad ①$$

$$35.2 = m \cdot (55°C) + b \qquad ②$$

Subtracting ② - ① gives: $5.2 = m \cdot 35C°$

$$\Rightarrow m = 0.15\,(C°)^{-1}$$

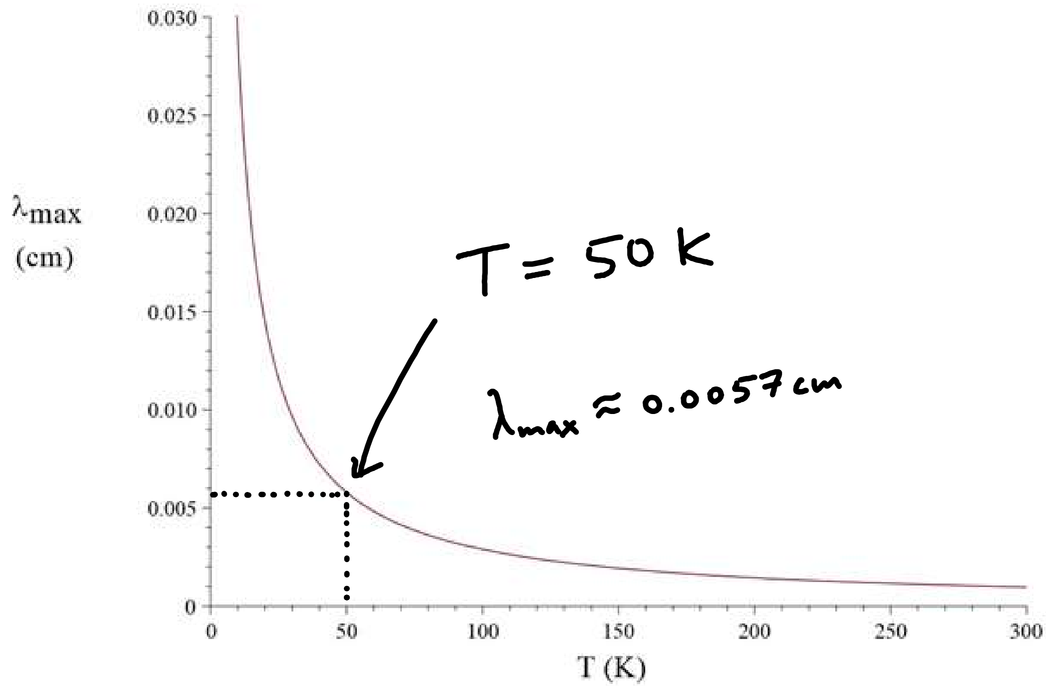
Using this in ① gives $b = 30 - 20°C \cdot m = 27$.

At 30°C, we then have:

$$G = m \cdot (30°C) + b = (0.15) \cdot 30 + 27 = \underline{\underline{31.5}}$$

② We are given that $\lambda_{max} = \frac{C}{T}$ for some constant C.

To find C, we can take a data point from the graph:



We get $C = \lambda_{max} \cdot T \approx 0.0057\text{cm} \cdot 50\text{K} = 0.28 \text{ cm}\cdot\text{K}$

Then for $\lambda_{max} = 0.107\text{cm}$, we get $T = \frac{C}{\lambda_{max}}$

$$= \frac{0.28\text{cm}\cdot\text{K}}{0.107\text{cm}}$$

$$\approx \underline{\underline{2.6\text{K}}}$$

This radiation (whose wavelength corresponds to microwaves) is called the cosmic microwave background radiation. It is the radiation left over from very early in the universe when atoms first formed and the universe became transparent (before that, the whole universe was like the interior of the sun).

③ a) We are given that the resistance is

$$R = R_0 (1 + A \cdot T + B \cdot T^2)$$

where T is the temperature in degrees Celcius.

At the ice point of water, $T = 0°C$ and $R = 10.000$ ohms, so $\underline{R_0 = 10.000}$. Using the other data provided, we get:

$$13.946 = 10.000\left(1 + A \cdot 100°C + B \cdot (100°C)^2\right) \quad ①$$

$$24.174 = 10.000\left(1 + A \cdot (419.514°C) + B \cdot (419.514°C)^2\right) \quad ②$$

From ①, we get $B = \dfrac{0.3946}{(100°C)^2} - \dfrac{A}{100°C}$ ③

Plugging this into ② and solving for A, we get:

$$\underline{A = 0.004123\ (C°)^{-1}}$$

From ③, $\underline{B = -1.776 \times 10^{-6}\ (C°)^{-2}}$.

b) If $R = 17.7$ ohms, we have:

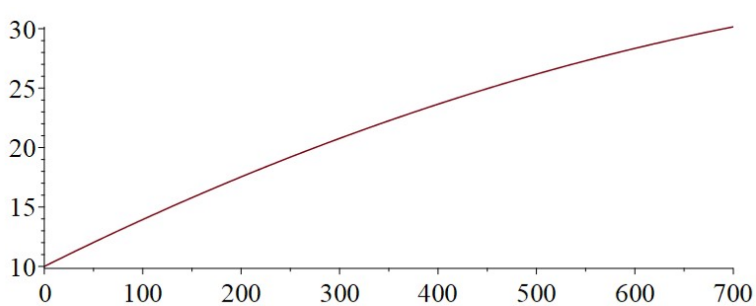
$$17.4 = 10.0\left(1 + AT + BT^2\right)$$

so: $BT^2 + AT - 0.74 = 0$

$$\Rightarrow T = -\frac{A}{2B} \pm \frac{1}{2B}\sqrt{A^2 + 4 \cdot B \cdot 0.74} = 204.8°C,\ 2117°C$$

Our equation is only valid between 0°C and 700°C, so the temperature should be 204.8°C.

c) plot: R (ohms)



T(K)

(a plot by hand is also fine)