Physics 157 Tutorial – week of October 21

This tutorial gives you more practice with ideal gas processes. This week, we'll focus more on heat and the change in internal energy, and introduce the idea of calculating the efficiency of a heat engine.

You can check your answers during the tutorial using Mastering Physics. Hand in your worksheet at the end.

Useful formulae and tips:

Important formulae and tips:

Ideal gas law: \[ P \, V = n \, R \, T \] or \[ \frac{P_1 \, V_1}{T_1} = \frac{P_2 \, V_2}{T_2} \] if \( n \) is constant.

Work: \[ W = P \, \Delta V \] (constant pressure) or area under curve on P-V diagram. Positive for expansion, negative for compression.

Internal energy: \[ \Delta U = n \, C_v \, \Delta T \]

Heat: \[ Q = \Delta U + W \] (First Law of Thermodynamics)

Special processes:

**Constant volume:**

T/P is constant, \( W = 0 \) so \( Q = \Delta U \)

**Constant pressure:**

T/V is constant, \( W = P \, \Delta V \), \( Q = n \, C_p \, \Delta T \) where \( C_p = C_v + R \)

**Constant temperature:**

PV is constant, \( \Delta U = 0 \), so \( Q = W \). \( W = n \, R \, T \, \ln(V_f/V_i) \),

**Adiabatic:**

Q = 0, \( P \, V^\gamma = \) constant, \( T \, V^{\gamma-1} = \) constant where \( \gamma = \frac{C_p}{C_v} \), \( W = -\Delta U \)
Problem 1: From an old midterm:

3. A cylinder contains Nitrogen gas \( (C_v = 20.8 \text{J/(mol K)}, \gamma = 1.4) \) at a pressure of 2.0 atm and a temperature of 300K. Its initial volume is 2.0 liters. The Nitrogen gas is next carried through the following processes: 1) It is heated at constant volume to a temperature of 450K. 2) It is expanded at constant temperature to a volume of 3.0 liters. 3) It is compressed at constant pressure until its volume is 2.0 liters, returning it to its initial state.

a) Draw the PV curve of this cycle. Be sure to label the temperature, pressure, and volume at each endpoint.

b) What is the efficiency of this heat engine?

Note: efficiency of the engine is defined to be the net work done during the cycle divided by the heat added, ignoring any heat that flows out (i.e. parts where \( Q < 0 \))

See the last page for hints and a step-by-step approach, but see how much you can do without using this!
**Problem 2** An ideal gas with $C_V = 3R$ is in a cylinder with area 0.25cm$^2$ with a piston that is originally 15cm above the bottom of the cylinder. The gas is initially at room temperature ($20^\circ$C) and atmospheric pressure (100kPa). A 1.2kg weight is placed on the top of the piston causing it to compress very quickly. At maximum compression, the piston is only 1cm above the bottom of the cylinder. What is the final temperature of the gas? You can ignore the mass of the piston and the gas, and ignore work done by the outside air.

*See the last page for hints and a step-by-step approach, but see how much you can do without using this!*

**Problem 3** For the cycle in the question 1), suppose that we are supplying heat by burning gasoline (this releases 34MJ/L), and we want to produce 1000W of mechanical power. What will be the required rate of fuel consumption in liters per hour?

If you have extra time, discuss the HW5 bonus problem (in the Written Problems section of Canvas) with your group.
Problem 1 hints: (check answers at the bottom of the page)

Hints for part a): ideal gas law!

i) What is the pressure after step 1?

ii) What shape is the constant temperature part?

Hints for part b): see the tips for special processes on page 1

iii) What is the work done by the gas for step 1?

iv) What is the work done by the gas for step 2?

v) What is the work done by the gas for step 3?

vi) What is the net work done for the cycle?

vii) How many moles of gas are there?

viii) What is ΔU for step 1?

ix) What is ΔU for step 2?

x) What is ΔU for step 3?

xi) What is Q, the heat added for step 1?

xii) What is Q, the heat added for step 2?

xiii) What is Q, the heat added for step 3?

xiv) How much heat flows into the gas during the cycle (ignore parts where heat flows out)?

xv) What is the efficiency?

Problem 2 hints: (check answers at the bottom of the page):

i) What type of process is this (hint: it happens very quickly)?

ii) What is the change in potential energy of the weight during this process?

iii) What is W for the gas?

iv) What is the change in internal energy?

v) How many moles of gas are in the cylinder?

vi) What is the temperature change?

Answers to the questions above, not in order:

-700, -500, -200, -1.65, 0, adiabatic, 0, 0.000154, 0.160, 0.058, 1.65, 1.65, 1.82, 3.00, 43, 243, 243, 430, 500, 500, 743