Learning goals:

• Give an example of a material where the thermal expansion coefficient has a significant temperature dependence and describe some consequences of this
• Define the concepts of stress and strain and explain how the Young’s modulus of a material is defined
• To argue that the Young’s modulus depends only on the material and not on the shape or size
• To calculate the fractional change in length of an object given its dimensions, the Young’s modulus, and the forces applied
• For static systems with multiple parts, to use Newton’s second and third law to related the various internal forces
• to calculate the mechanical forces (thermal stress) on a constrained system arising from changes in temperature.
Homework sessions with TAS:
  Monday 5-7pm in Hennings 200
  Tuesday 5-7pm in Hennings 202

Piazza: an online Q&A Forum for questions about homework, course material, etc...

Office hours: Today: 3-5 pm
  Hennings 420
Last time in Physics 157...
Thermal expansion:

\[ \Delta L = \alpha L_0 \Delta T \]

applies to each dimension

\[ \Delta V = \beta V_0 \Delta T \]

\[ \beta = 3\alpha \text{ for solids} \]
Water, a special example

While the expansion of water with temperature is *approximately* linear ...

... on a fine scale it is not exactly linear, which has important consequences for life on earth.

Water is most dense at 4°C.

$\beta$ positive

$\beta$ negative

$\beta = 0$
Today: thermal expansion

forces
**Stress & Strain**

\[
\frac{F}{A} = Y \frac{\Delta l}{l_0}
\]

- **Stress** (units of pressure)
  - **Young's modulus**
    - a basic property of a material (resistance to squishing)
  - \( F = k\Delta x \) for spring

**Compression**

\[ l_0 + \Delta l \]

**Tension**

\[ l_0 + \Delta l \]
Young's modulus of a marshmallow

\[ \frac{F}{A} = Y \frac{\Delta L}{L}. \]

Mean \( \approx 8 \times 10^3 \text{ N/m}^2 \)
Young’s modulus of a marshmallow

\[
\frac{F}{A} = Y \frac{\Delta L}{L_0}
\]

\[
\frac{\Delta L}{L_0} \approx 0.1 - 0.2
\]

\[
F \approx 1 \text{ N}
\]

\[
A \approx 5 \text{ cm}^2 = 5 \times (10^{-2} \text{ m})^2
\]

gives \( Y \approx 10^4 \text{ N/m}^2 \)
Clicker: Suppose you repeated the measurement of Y for a mini-marshmallow. In this case, we would expect a value of Y that is

A. Significantly higher
B. Significantly lower
C. About the same

\[ \frac{F}{A} = Y \frac{\Delta L}{L_0} \]
Clicker: In the top picture, the force on the right brick from the left brick has magnitude

A) 0
B) F
C) 2F

All forces shown have the same magnitude.

EXTRA: can you come up with a sharp argument for your answer?
**Clicker:** In the top picture, the force on the right brick from the left brick has magnitude

A) 0

B) F

C) 2F

All forces shown have the same magnitude.

Each half above has same compression as this brick.
\[ Y = \frac{F/A}{\Delta L/L_0} \]

- **Same F, A:**
  - double \( L_0 \), double \( \Delta L \)
  - \( \therefore \) same \( Y \)

- **Double F, double A:**
  - same \( \Delta L, L \)
  - \( \therefore \) same \( Y \)

\( Y \) is the same in all cases
\( \therefore \) it only depends on the material
# Young Modulus of Various Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus, $Y$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$7.0 \times 10^{10}$</td>
</tr>
<tr>
<td>Brass</td>
<td>$9.0 \times 10^{10}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$11 \times 10^{10}$</td>
</tr>
<tr>
<td>Crown glass</td>
<td>$6.0 \times 10^{10}$</td>
</tr>
<tr>
<td>Iron</td>
<td>$21 \times 10^{10}$</td>
</tr>
<tr>
<td>Lead</td>
<td>$1.6 \times 10^{10}$</td>
</tr>
<tr>
<td>Nickel</td>
<td>$21 \times 10^{10}$</td>
</tr>
<tr>
<td>Steel</td>
<td>$20 \times 10^{10}$</td>
</tr>
<tr>
<td>Marshmallow</td>
<td>$8 \times 10^{3}$</td>
</tr>
</tbody>
</table>

Units of pressure: $1 \text{ Pa} = 1 \text{ N/m}^2$

Example: Pressure of 0.01% of $Y$ on ends will give 0.01% compression.

\[
\frac{F}{A} = Y \frac{\Delta L}{L}
\]
Elastic vs. Plastic

stress

strain

Elastic regime

linear range

plastic deformation

irreversible change past this point

fracture point

\[ \frac{F}{A} = Y \frac{\Delta L}{L_0} \]

valid here
THERMAL STRESS: forces on a material due to surrounding materials preventing thermal expansion/contraction
A steel rod of length $L_0$ is heated by temperature $\Delta T$.

How much stress (force per unit area) is required to compress the rod back to its original length?

Write an answer in terms of $\Delta T$ and the parameters $\gamma$, $\alpha$, $L_0$ for the rod.

Click A if you have an answer, B if you are stuck.

**thermal expansion:**

$$\Delta L = \alpha L_0 \Delta T$$

**stress vs strain:**

$$\frac{F}{A} = \gamma \frac{\Delta L}{L_0}$$
A steel rod of length \( L_0 \) is heated by temperature \( \Delta T \) and expands. How much stress (force per unit area) is required to compress the rod back to its original length?

Write an answer for the magnitude of \( F/A \) in terms of \( Y, \alpha, L_0, \) and \( \Delta T \).

A) \( Y \alpha L_0 \Delta T \)

B) \( Y \alpha \Delta T \)

C) \( Y L_0 \Delta T \)

D) \( \alpha L_0 \Delta T \)

E) \( Y \alpha L_0 \)

\[ \Delta L_{th} = \alpha L_0 \Delta T \]

want \( \Delta L_F = -\Delta L_{th} = -\alpha L_0 \Delta T \)

\[ \frac{F}{A} = Y \left( \frac{\Delta L_F}{L_0} \right) \]

\[ \frac{F}{A} = Y (-\alpha \Delta T) \]

\[ \left| \frac{F}{A} \right| = Y \alpha \Delta T \]
Clicker: 10m long steel train rails are laid end to end on a winter day (0 ºC). If the engineer forgot to leave gaps for thermal expansion, roughly how much force is generated at the ends of each rail due to thermal stress when the temperature reaches 30 ºC?

Cross sectional area of rail: 0.01m$^2$

$Y_{steel} = 20 \times 10^{10}$ Pa

$\alpha_{steel} = 1.2 \times 10^{-5}$ K$^{-1}$

A) 700 N  B) 7,000 N  C) 70,000 N  D) 700,000 N  E) 7,000,000 N

EXTRA: How much gap should have been left?
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$$F \approx 10^{-2} \times 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 30 \approx 7 \times 10^5$$

EXTRA: How much gap should have been left?  

$\Delta L = \alpha L \Delta T \approx 3.6 \text{ mm}$
Extra Clicker: Do you expect that the Young’s modulus you measured for a marshmallow is higher or lower than for steel?

A. Higher
B. Lower
C. Could be higher or lower depending on the relative dimensions of the steel/marshmallow

\[
\frac{F}{A} = Y \frac{\Delta L}{L_0}
\]
Clicker: Do you expect that the Young’s modulus you measured for a marshmallow is higher or lower than for steel?

A. Higher
B. Lower
C. Could be higher or lower depending on the relative dimensions of the steel/marshmallow

$Y$ only depends on what the object is made of, not its size.

$\frac{F}{A} = Y \frac{\Delta L}{L}$: $Y$ bigger if it takes more force to give same change in length.

$Y$ has units of pressure: roughly, the pressure required to produce a significant fractional change in length.