Learning Goals:

• For an object made of some material, to calculate the changes in length or volume that material undergoes in response to changes in temperature and external forces (stress).
• To explain why the change of length of an object due to thermal expansion is proportional to its initial length.
• For systems consisting of two different materials, to quantitatively analyze effects resulting from the different expansion rates of different parts.
• To explain why the fractional change in volume of an object for a small change in temperature is three times the fractional change in length.
Clicker: A steel ball does not quite fit through a hole in a copper plate. If $\alpha_{\text{steel}} < \alpha_{\text{copper}}$, we could help the ball fit through the hole by

A. Heating the system
B. Cooling the system
C. Either A or B will work
D. Neither A nor B will work

EXTRA: does the hole get larger or smaller when we heat the system? Why?
Define Kelvin scale by:

\[ T = \text{const.} \times \frac{\text{const.}}{\text{pressure}} \]

and

\[ T = 273.16 \text{ K} \]

at triple point of water

\[ T_c = T_k - 273.15 \]
Thermal expansion:

\[ \Delta L = \alpha L_0 \Delta T \]

- Assumes \( \frac{\Delta L}{L} \) is small
- \( \alpha \) can depend on \( T \)

Coefficient of linear expansion: a basic property of a material
Discussion question: why is the change in length of an object proportional to its initial length $L_0$? E.g. why does a steel rod that starts out twice as long expand twice as much?

$$\Delta L = \alpha L_0 \Delta T$$
Discussion question: why is the change in length of an object proportional to its initial length $L_0$? E.g. why does a steel rod that starts out twice as long expand twice as much?

Each half expands independently, same amount as original object. Total expansion is double.

$$\Delta L = \alpha L_0 \Delta T$$
Why do materials usually expand when heated?

Nearby atoms in solid:

More higher energy configurations with $r > r_0$ than $r < r_0$. 

Potential Energy

Lowest energy

Add energy
Clicker: A steel ball does not quite fit through a hole in a copper plate. If \( \alpha_{\text{steel}} < \alpha_{\text{copper}} \), we could help the ball fit through the hole by

A. Heating the system
B. Cooling the system
C. Either A or B will work
D. Neither A nor B will work

EXTRA: does the hole get larger or smaller when we heat the system? Why?
**Clicker:** A steel ball does not quite fit through a hole in a copper plate. If $\alpha_{\text{steel}} < \alpha_{\text{copper}}$, we could help the ball fit through the hole by

- A. Heating the system
- B. Cooling the system
- C. Either A or B will work
- D. Neither A nor B will work

**EXTRA:** does the hole get larger or smaller when we heat the system? Why?
If the radius of the ball at $T = 20^\circ C$ is 1.001cm and the radius of the hole is 1.000cm, to what temperature must we heat the system before the ball falls through?

We have: $\alpha_s = 1.2 \times 10^{-5} \text{ K}^{-1}$ and $\alpha_c = 8 \times 10^{-5} \text{ K}^{-1}$

Discuss a strategy for solving this. What should be true about $\Delta L_{\text{ball}}$ relative to $\Delta L_{\text{hole}}$?

$\Delta L = \alpha L \Delta T$
Strategy:

1. Understand what happens to each part

2. Understand how the parts are related

1. Hole expands: \[ \Delta L_{\text{hole}} = \alpha_{\text{Cu}} L_0 \Delta T \] (unknown)

2. Ball expands: \[ \Delta L_{\text{ball}} = \alpha_{\text{s}} L_0 \Delta T \]

2. We need \( \Delta L_{\text{hole}} = \Delta L_{\text{ball}} + 0.001 \text{cm} \)
   in order for the ball to fall through

3. Rest is math: \[ \alpha_{\text{Cu}} L_0 \Delta T = \alpha_{\text{s}} L_0 \Delta T + 0.001 \text{cm} \]
   solve for \( \Delta T \)
Clicker: In some car engines, the piston is aluminum ($\alpha = 2.4 \times 10^{-5}$), while the cylinder is cast iron ($\alpha = 1.2 \times 10^{-5}$). If the engine needs to operate between $0^\circ C$ and $120^\circ C$, which of these is not a good design:

A) The piston barely fits in the cylinder at $120^\circ C$

B) The piston barely fits in the cylinder at $0^\circ C$

EXTRA: what do we need to worry about if the engine gets too hot? Too cold?

$$\Delta L = \alpha L \cdot \Delta T$$
Clicker: In some car engines, the piston is aluminum \( (\alpha = 2.4 \times 10^{-5}) \), while the cylinder is cast iron \( (\alpha = 1.2 \times 10^{-5}) \). If the engine needs to operate between 0ºC and 120ºC, which of these is not a good design:

A) The piston barely fits in the cylinder at 120ºC

B) The piston barely fits in the cylinder at 0ºC

\[ \Delta L = \alpha L \cdot \Delta T \]

EXTRA: What do we need to worry about if the engine gets too hot? Too cold?
Volume expansion:

\[ \Delta V = \beta V \Delta T \]

also applies to liquids
Clicker: When heated, each side of a 1m cube of material expands by 0.001m. The extra volume (shown in the third picture) after the expansion is approximately

A) 0.0000000001m$^3$  B) 0.00001m$^3$  C) 0.001m$^3$  D) 0.003m$^3$

E) There is not enough information

*look at the picture and use geometry to solve this!*
Clicker: When heated, each side of a 1m cube of material expands by 0.001m. The extra volume (shown in the third picture) after the expansion is approximately

A) 0.000000001m³  B) 0.00001m³  C) 0.001m³  D) 0.003m³  E) There is not enough information

E) There is not enough information

Look at the picture and use geometry to solve this!
Volume expansion:

\[ \Delta V = \beta V_0 \Delta T \]

\[ \beta = 3\alpha \text{ for solids} \]

also applies to liquids
Mathematical derivation:

original volume: \( L^3 \)
new volume \((1.001 \times L)^3 \approx 1.003 \ L^3\)

so 0.3% bigger

generally: \((L + \Delta L)^3 = L^3 + 3 \ L^2 \Delta L + 3 \ L (\Delta L)^2 + (\Delta L)^3\)

\[
\frac{\Delta V}{V} = 3 \cdot \frac{\Delta L}{L} + 3 \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta L}{L}\right)^3
\]

this means \( \beta = 3 \alpha \)

these are negligible compared to the first term if \( \frac{\Delta L}{L} \) is small