Learning Goals

• Calculate the equilibrium temperature of a radiating object (e.g. the Earth) by equating the ingoing and outgoing energy currents
• Describe what is meant by intensity of radiation
• Calculate the intensity of radiation at some distance from an object radiating symmetrically in all directions
• Describe how the intensity of radiation changes if we change the distance from the source, for a source radiating uniformly in all directions
• Predict the rate of energy absorbed by an object given its shape and orientation, its albedo, and the intensity of incident radiation
• Explain why the presence of greenhouse gases in the atmosphere of Earth lower its effective emissivity
Last time in Physics 157...
Total Power from Thermal Radiation

\[ H = A \cdot e \cdot \sigma \cdot T^4 \]

- **Surface area**
- **Heat current**
- **Emissivity**
- **Stefan-Boltzmann constant** $5.67 \times 10^8 \, \text{W} / \text{m}^2 \cdot \text{K}^4$

- $e = 1$ perfect absorber (black)
- $e = 0$ perfect reflector (mirror)
NET HEAT CURRENT FROM THERMAL RADIATION

(in uniform temperature environment)

surface area

$H_{\text{rad}} = A \cdot e \cdot \sigma \cdot T^4$

$H_{\text{absorbed}} = A e \sigma \cdot T_{\text{surroundings}}^4$

equilibrium: $H_{\text{absorbed}} = H_{\text{rad}}$
Key relation for steady-state heat flow:

\[ H_{in} = H_{out} \]

\[ H_{in} \xrightarrow{\text{const } T} H_{out} \]
Temperatures not changing

\[ H_{in} = H_{out} \text{ (for planet outside core)} \]

\[ P_{heater} = A \sigma e T^4 \]

\[ T_{surface} = \left( \frac{P_{heater}}{A \sigma e} \right)^{\frac{1}{4}} \]
A more interesting one...

A planet with radius \( r = 6400 \text{km} \) lies at a distance \( R = 150,000,000 \text{km} \) from a yellow star with temperature \( T = 5700 \text{K} \) and radius \( R_S = 695,000 \text{km} \). Estimate the surface temperature of the planet.

The planet has **albedo** (fraction of incident light reflected) \( A = 0.37 \) and emissivity \( e \) close to 1.
Key relation for steady-state heat flow:

\[ H_{in} = H_{out} \]

Our problem:

What is \( H_{in} \)?

\( H_{in} \): absorbed sunlight

\( H_{out} \): IR radiation

\( H_{out} = Ae \sigma T^4 \)
A gigantic sphere with radius R is built surrounding the sun. A hole is cut into the sphere, removing an area A. What is the rate of energy flow for light from the sun through the hole in terms of the Sun’s total power $H_{\text{sun}}$?

A) $H_{\text{sun}} \times A$

B) $H_{\text{sun}} \times \frac{A}{R^2}$

C) $H_{\text{sun}} \times \frac{A}{\pi R^2}$

D) $H_{\text{sun}} \times \frac{A}{4 \pi R^2}$

**EXTRA:** If $H_{\text{sun}} = 3.86 \times 10^{26}$ W and $R = 1.5 \times 10^{11}$ m, how much solar energy per second goes through an area of 1 m$^2$ at the distance $R$?
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- Light spreads out uniformly
- Power leaving sun = power reaching sphere
- Hole covers fraction $\frac{A}{4\pi R^2}$ of sphere
- So power of light coming out is $\frac{A}{4\pi R^2} \times H_{\text{sun}}$
The solar constant.

- The power from the Sun is \( H_{\text{Sun}} = A_{\text{Sun}} \cdot \sigma \cdot T_{\text{Sun}}^4 \).

- At Earth's orbit, the power per unit area (or intensity) of sunlight is

\[
I_{\text{sc}} = \frac{H_{\text{S}}}{4\pi R_{\text{Sun}}^2} = 1367 \text{ W/m}^2
\]

"solar constant"
What is the power $H_{\text{in}}$ of solar radiation absorbed by the Earth? Answer in terms of $I_{\text{sc}}$, the albedo $a$ (fraction of sunlight reflected) and the Earth’s radius $r$.

\[ A) I_{\text{sc}} \cdot \pi r^2 \cdot a \quad B) I_{\text{sc}} \cdot \pi r^2 \cdot (1-a) \]

\[ C) I_{\text{sc}} \cdot 2\pi r^2 \cdot a \quad D) I_{\text{sc}} \cdot 2\pi r^2 \cdot (1-a) \]

\[ E) I_{\text{sc}} \cdot 4\pi r^2 \cdot (1-a) \]
Each block is the same area of sunlight: \( \pi r^2 \)
The heat current into the earth due to sunlight is\[ H_{in} = \pi r^2 (1-a) I_{sc} \]

Calculate the equilibrium surface temperature \( T \) in terms of \( a, I_{sc}, r, \sigma, \) and the emissivity \( e. \)

![Diagram showing solar radiation and IR radiation]

**Recall:** \[ H_{rad} = A e \sigma T^4 \]
The heat current into the earth due to sunlight is

\[ H_{in} = \pi r^2 (1-a) I_{Sc} \]

Calculate the equilibrium surface temperature T in terms of \( a \), \( I_{sc} \), \( r \), \( \sigma \), and the emissivity \( e \).
The heat current into the earth due to sunlight is \[ H_{\text{in}} = \pi r^2 (1-a) I_e \]

Calculate the equilibrium surface temperature \( T \) in terms of \( a, I_{\text{sc}}, r, \sigma \), and the emissivity \( e \).

\[
\text{H}_{\text{in}}: \text{absorbed sunlight} \\
\text{H}_{\text{out}}: \text{IR radiation}
\]

We have \( H_{\text{in}} = H_{\text{out}} \) (steady state)

\[
\pi r^2 (1-a) I_{\text{sc}} = 4\pi r^2 \cdot e \cdot \sigma \cdot T^4
\]

\[
T = \left[ \frac{(1-a) I_{\text{sc}}}{4e\sigma} \right]^\frac{1}{4}
\]
\[ T = \left[ \frac{(1-a)I_{sc}}{4e\sigma} \right]^\frac{1}{4} \]

The numbers: surface of the Earth has \( e \approx 1 \) for IR radiation.

\( I_c = 1367 \text{ W/m}^2 \quad a = 0.3 \quad \sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4} \)

These give \( T \approx -18^\circ\text{C} \)

Something is off...
Actual surface temperature is larger due to the **GREENHOUSE EFFECT**: some IR radiation is absorbed by “greenhouse gases” and re-emitted back to Earth.

we assumed: $$e = 1$$

actual: $$e \approx 0.6$$
\[ T = \left[ \frac{(1-a)I_{sc}}{4e\sigma} \right]^\frac{1}{4} \]

Lower \( e \) \( \Rightarrow \) higher \( T \)

Real \( e \approx 0.6 \) gives \( T = 14.5^\circ C \)

But \( e \) can decrease e.g. due to increasing \( CO_2 \) concentration in atmosphere. \( \rightarrow \) Global warming
$\text{CO}_2$ correlates closely with temperature.
Almost all climate scientists believe this rise due to human activity