

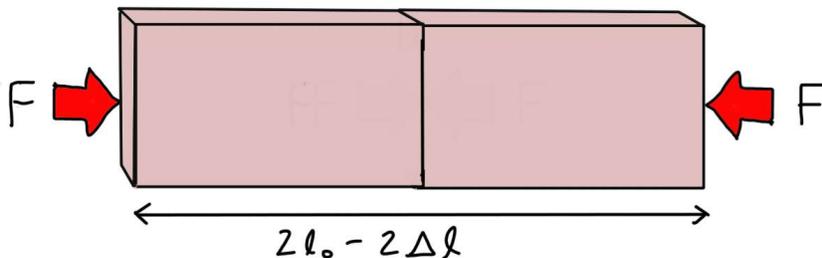
## Young's modulus:

We can compress or stretch a solid material by acting with opposing forces on opposite sides (see figure to the right). For small amounts of compression, force required to produce a certain change in length  $\Delta l$  is directly proportional to  $\Delta l$ , just like with Hooke's law for a spring. We have:

$$F = C \Delta l$$

The constant of proportionality  $C$  here depends on the type of material we are compressing but also on the dimensions of the object.

For a block of material, if we double the length, we'll get twice as much compression for the same force applied, since each half of the new object will experience the same forces as for our original object (in the picture below, there will also be "normal" forces in the middle which act to keep each side in place). So the constant  $C$  should half if we double  $l_0$ .



If we double the cross-sectional area of the block, we'll need twice as much force to get the same compression, since this situation is equivalent to having two of the original blocks side by side as shown. So the constant  $C$  should double.

Putting everything together, we can say that  $C$  should be proportional to  $A$  be inversely proportional to  $l_0$ , thus

$$C = Y A / l_0$$

Now, the proportionality constant  $Y$  is a basic property of the material itself that does not depend on the dimensions. We call this the **Young's Modulus** of the material – this. Combining our two equations, we can write the relation between force applied and the change in length as:

$$(F / A) = Y (\Delta l / l_0)$$

The left side here is the force per unit of cross-sectional area perpendicular to the area we are considering. It has the same units as pressure, and is called the **stress** in the material. The quantity  $(\Delta l / l_0)$  is the fractional change in length and is called the **strain** on the material. So we see that stress is proportional to the strain, and that Young's modulus gives the proportionality constant.

