Physics 157 Homework 8: due Wed, Nov 20th by 5pm

In this homework set, you'll get more practice analyzing systems in simple harmonic motion (SHM). Here are the skills that we'd like you to develop in addition to the ones from last week:

- Given a physical system with a variety of relevant forces, to write an expression for the net force on some object in the system as a function of that object's position, and to use this to deduce the equilibrium position and the frequency for oscillations around equilibrium
- Given the description of the initial configuration for a physical system that will undergo simple harmonic motion, to predict the subsequent motion of the object, representing this as a sinusoidal function.
- Given the description of the initial configuration for a physical system that will undergo simple harmonic motion, to predict the amplitude or maximum velocity using conservation of energy.

Your Homework: Do the three written questions below and hand them in to the homework box.

Restoring force leading to SHM: $\mathbf{F_{NET} = - k\, \Delta x}$

Finding k in a general system: a) determine F_{NET} as a function of position
b) find the equilibrium position via $F_{NET}(x_{eq}) = 0$
c) $-k$ is the slope of $F_{NET}(x)$ at x_{eq}, $k = -dF/dx$ at $x = x_{eq}$

Newton's 2nd Law: $\mathbf{a = F_{NET} / m}$

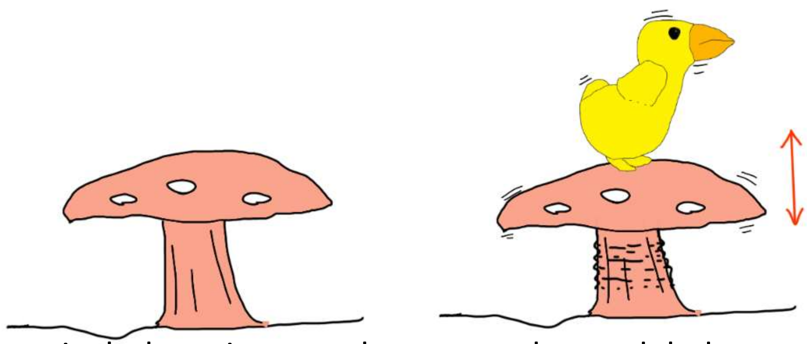
Basic form of simple harmonic motion: $\mathbf{x(t) = A \cos(\omega t + \phi)}$ $\mathbf{\omega^2 = k/m}$

Here A is the amplitude, ω is the angular frequency, related to the period by $\mathbf{\omega = 2\pi / T}$, and ϕ is the phase.

The phase can be determined looking at what fraction of a period the graph is shifted to the left or right by (take the time that the function is maximum and divide by T) , and then multiplying by ±2π, with + for shifts to the left and – for shifts to the right.

Velocity and acceleration: $\mathbf{v(t) = dx/dt}$ (slope of displacement graph at time t)

$\mathbf{a(t) = dv/dt}$ or $\mathbf{a = - \omega^2 x}$



Problem 1) A particularly springy mushroom can be modeled as mass of 30g on top of a spring with spring constant 20 N/m. A bird of mass 50g lands on the mushroom gently so that its velocity is zero when it lands.

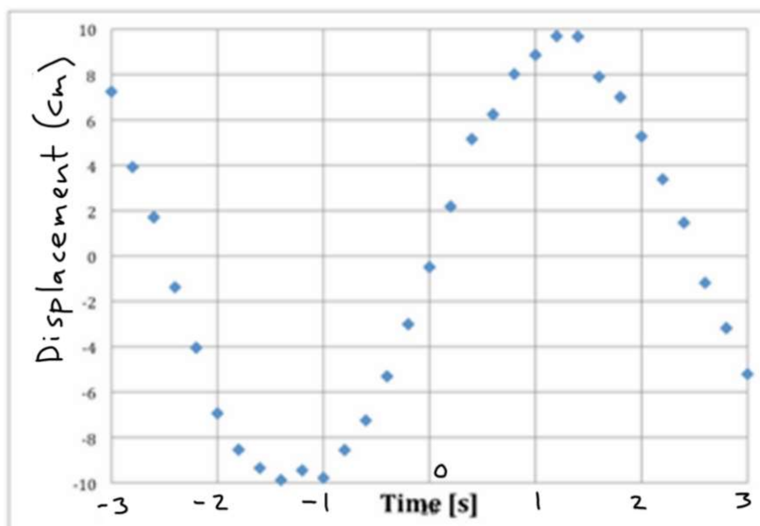
a) Relative to the initial height of the mushroom, what is the new equilibrium height of the mushroom about which oscillations will occur?

b) What is the frequency of the resulting oscillations?

c) What is the maximum compression of the mushroom?

d) What is the equation that describes the oscillation of the mushroom as a function of time?

Problem 2) The displacement as a function of time for a cylindrical buoy bobbing in the water is shown in the graph.



a) Write the displacement as a function of time, in the form $x(t) = A\cos(\omega t + \phi)$.

b) Approximately what is the maximum vertical speed of the buoy and at what time(s) does this occur in the time interval shown on the graph?

c) The net downward force on the buoy when it is at a depth h is given by

$F_{NET} = mg - A\,h\,g\,\rho_{water}$

where $A = 0.5m^2$ is the cross sectional area of the buoy and ρ_{water} is the density of water.[1] What is the mass of the buoy? *Hint: what is k for this system?*

d) What is the net upward force on the buoy when it is deepest in the water?

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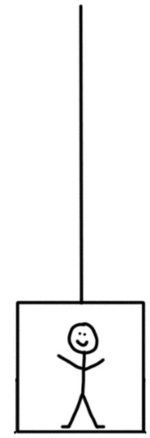
[1] Here, the second force is the **buoyant force** on the object, which is the net force on a submerged or partly submerged object from the water and the surrounding air. It is equal to the weight of the water displaced by the object, i.e. the volume of the part of the object that is under water times the density of water, times g.

Problem 3) The diagram shows an elevator with a cable of normal unstretched length 50m, Young's modulus 200GPa, and diameter 1cm.

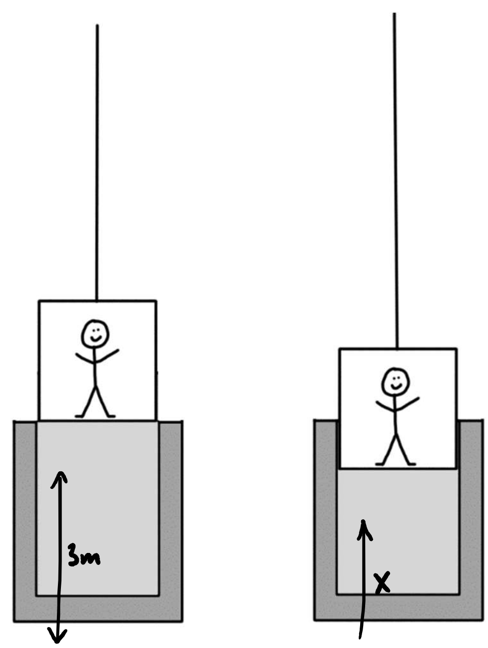


a) What is the spring constant for this cable?

Hint: the spring constant is defined by $F = - k \Delta L$. You want to find k in terms of the quantities given based on the definition of Young's modulus.

b) If the elevator plus passenger has a total mass of 500kg, what is equilibrium length of the cable and what is the oscillation frequency around this equilibrium point?

Suppose that as a safety feature, the bottom of the elevator shaft acts as a cylinder of gas that is compressed adiabatically when the cable stretches longer than its normal length, as shown in the picture (first picture is with the cable unstretched, second picture is the equilibrium position). The area of the elevator bottom is 1m^2 and the depth of the cylinder is 3m. The outside air pressure and the uncompressed pressure of the gas in the cylinder are 100kPa.



c) Defining x to be the height of the elevator above the bottom, find the net upward force on the elevator as a function of x. *(Hint: there are four forces you need to consider)*

d) Sketch a graph of this net force as a function of x for positive values of x. Indicate the equilibrium position of the elevator on your graph

e) What is the equilibrium value of x? *Hint: you won't be able to solve for the height by hand. You can get it by graphing, or using a calculator or Wolfram Alpha (www.wolframalpha.com: type in something like "solve x^2 + 1/x – 7 for x".)*

f) What is the new oscillation frequency about this equilibrium point?

Hint: for parts d and e, see the tips on the first page about finding k in a general system.

Extra practice problems from old exams (not to be handed in):

Problem 5. An unnamed superhero with spider-enhanced powers leaps off the Lions gate bridge to test a new spider silk. He falls completely vertically with no initial velocity a distance of 61m from the bridge to the water below. The spider silk material is quite elastic and he oscillates at the end of the fibre. He has spun exactly the right amount of fiber to just touch the water when he reaches the lowest point in the motion. He is wearing a very low friction suit such that damping effects can be neglected. He has recorded data about the jump.

(a) The superhero finds that he has a natural oscillation frequency corresponding to a period of 1.0s at the end of the fibre. His mass is 75kg. What is the spring constant of the spider-silk fibre?

(b) How much does the spider-silk fibre stretch to reach the water? What is the unstretched length?

(c) How far beneath the bridge is the superhero's equilibrium position?

(d) Where is our superhero 0.5s after first reaching the equilibrium position? In which direction is he travelling (towards the water or towards the bridge deck)?

(e) What was the maximum velocity of our superhero?

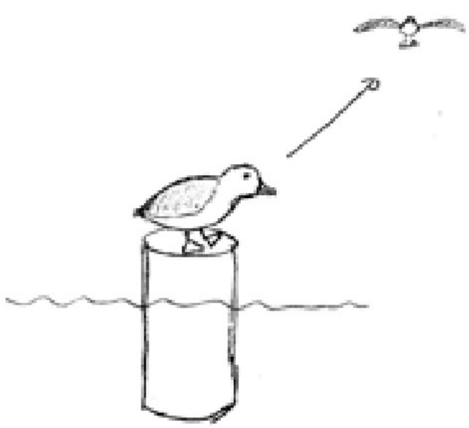
Problem 5. A seagull is sitting on top of a cylindrical buoy that weighs 30kg (as shown in the sketch). The buoy bobs up and down 10 times (complete oscillation cycles) in 6.4 seconds with the seagull sitting on top shortly after a wave passes. Once the buoy is stationary again, the seagull takes off gently (without exerting any significant downward force on the buoy by just flapping its wings). You observe that the buoy is now bobbing up and down 10 times in 6.3 seconds. In the following calculations, neglect damping effects (i.e. assume b is very small).

a) Sketch the vertical displacement from equilibrium as a function of time, $x(t)$, of the buoy from the time when the seagull leaves the buoy.

b) What is the mass of the seagull?

c) What is the amplitude of the motion of the buoy after the seagull takes off (note that the buoy had stopped moving before this)?

d) What is the maximum velocity of the buoy after the seagull takes off? And at what times after the seagull takes off does this occur?



Problem 5. You want to construct and analyze a model for magnetic levitation, so you build a device to hold a magnet directly above another one. Your structure holds the bottom magnet still, and prevents the top magnet from moving in the horizontal direction (x- and y-directions) but allows movement in the up and down (z-direction), so that the top magnet "floats" due to the magnetic repulsion.

For two identical magnets, the repulsive magnetic force in this setup is given by $F_{magnetic} = A/z^4$ where z is the distance between the magnets and A is constant. The constant A is determined by the strength of the magnets and their geometry.

Recall the Taylor polynomial expression for x near x_0 is $f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2!}f^{(2)}(x_0)(x - x_0)^2 + \ldots$.

(a) Draw the free body diagram and write down the algebraic expression for the net force on the top magnet.

(b) If $A = 1.2 \times 10^{-4} N\, m^4$, and each magnet has a mass of 100g, find the equilibrium distance between magnets for this device.

(c) If you push down and release the top magnet, you notice that it oscillates up and down. Find the effective spring constant and frequency for small oscillations of the top magnet using values from part (b).

(d) You begin a new oscillation by pushing the top magnet 0.5 cm down from its equilibrium position. Find the amplitude and period of the motion. Sketch a graph of displacement versus time for this magnet for the first 2 cycles of this oscillation, labelling the important points on the axes.