Computational Physics
Physics 410 2014W
Final Exam Project Questions
Due: Friday, December 12, 2014 11PM

Instructions: Please hand in your question both on the PHAS server as well as by emailing your code and answer PDF to **physics.410.ubc@gmail.com** . You must include a file **solution.pdf** that documents your solutions and any material/plots requested, your code in a file **code.pdf**, as well as your code in its native language.

Please Attempt All Questions if Possible.
The question you score lowest in will be weighted 50% relative to the others.

Problem 1

IMPORTANT: For this problem only, be sure to properly typecast all your results to 32bit floats using numpy.float32 if using python.

The Riemann Zeta Function can be defined through the sum:

$$\zeta(s) \equiv \sum_{n=1}^{\infty} \frac{1}{n^s}$$

At s=2 we have the analytic result:

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

We define the partial sum:

$$S_N = \sum_{n=1}^{N} \frac{1}{n^2}$$

a) With 32bit floating point arithmetic (only), compute S_N between $N = 10$ and $N = 10^7$ by summing upwards from $n = 1$ to $n = N$ and downwards from $n = N$ to $n = 1$. Plot the fractional error of your results as a function of $h = 1/N$ and explain the observed behaviour including the approximate scaling with h and the smoothness of the error as a function of h.

b) Assuming the error $E_N \equiv S_\infty - S_N = \frac{\pi^2}{6} - S_N$ scales like $O(h) = O(1/N)$, repeat part a) using Richardson extrapolation of your method S_N with a scale parameter $k = 2$. Plot the fractional error of your results as a function of $h = 1/N$ and explain the observed behaviour including the approximate scaling with h and the smoothness of the error as a function of h.

Problem 2

The first order correction to the ground state energy of a particle confined to the interval $0 < x < L$ is given by:

$$\delta E_1 = \frac{2}{L}\int_0^L dx \sin^2\left(\frac{\pi x}{L}\right)\delta V(x)$$

for some potential perturbation $\delta V(x)$. For the rest of the problem take:

$$\delta V(x) = \frac{e^{-10\pi^2 x/L}}{10 + (x/L)}$$

a) Compute δE_1 using the Trapazoidal, Simpson, and Gauss-Legendre methods with $N = 2^n$ points. Plot your estimate of δ_{E1} as a function of n from $n = 1$ $(N = 2)$ to $n = 9$ $(N = 512)$. Comment on the convergence of your results.Which method converges most quickly and why?

b) Compute δE_1 using Romberg Integration. How many Romberg iteration levels does it require to achieve a fractional tolerance of 10^{-3}? 10^{-6}? 10^{-9}?

Problem 3

The dimensionless gravitational 2-body problem (in the limit that one mass is much heavier than the other) can be written in the form:

$$\frac{d^2x}{dt^2} = -\frac{x}{r^3} \qquad \frac{d^2y}{dt^2} = -\frac{y}{r^3}$$

where

$$r^2 = x^2 + y^2$$

Express this system of two coupled second order equations as a system of four coupled first order equations. For initial conditions $x = 0.25$, $y = 0$, $v_x = 0$, $v_y = 1$ solve this system using the 4^{th} order Runge-Kutta method with stepsize h for $h = 0.01$, $h = 0.001$, and $h = 0.0001$. What are the initial energy and angular momentum of this system? For the time period $t = 0$ to $t = 1$ plot the trajectory of the particle and the change in the energy and angular momentum (relative to the initial value) as a function of time. Explain the behaviour you observe. At what point in the orbit to the energy and/or angular momentum change the most and why?

Problem 4

For this problem please use the flexible ODE integrator you designed for Assignment 4 for all solutions. You may refer to the specifications in Assignment 4 if needed. Please also download the **benchmark.py** python code snippets that define Benchmark Problem 1 (3-body problem), and Benchmark Problem 2 (damped forced pendulum).

a) Compute the solution to Benchmark Problem 1 using the "rk2", "leap", and "bs" methods. Present the results in terms of the number of function calls, a plot of the trajectory, and the values of the variables at the final time.

b) Compute the solution to Benchmark Problem 2 using the "euler", "mm", and "rk4adaptive" methods. Present the results in terms of the number of function calls, a plot of the trajectory, and the values of the variables at the final time.

Problem 5

Setting $\delta = 10^{-4}$, a grid with 120x120 interior points, and using a method of your choice solve in a square box the Poisson equation

$$\nabla^2 \phi = \rho$$

for the potential in the presence of a charged cylindrical annulus with an inner diameter one quarter the size of the box and an outer diameter one half the size of the box. Assume the outer box is grounded to $\phi = 0$. The annulus has a quadrapolar charge distribution with a charge $\rho = 1$ on the top left and bottom right quadrants and a charge $\rho = -1$ on the top right and bottom left quadrants. Plot both $\rho(x, y)$ and$\phi(x, y)$ as coloured density plots with a legend. Qualitatively describe the shape of $\phi(x, y)$ compared to the shape of $\rho(x, y)$, and explain this behaviour.