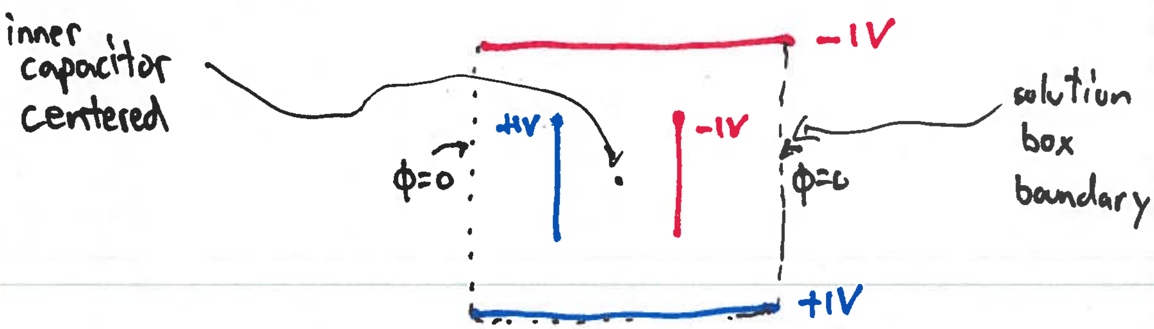
Computational Physics
Physics 410 2014W
Assignment 5
Due: Thursday, November 20, 2014 11PM

Design and code a series of PDE solvers that can solve the Poisson equation

$$\nabla^2 \phi = \rho$$

using the Jacobi, Gauss-Seidel, and Red-Black relaxation methods on a square cartesian grid. You should be able to specify an absolute tolerance parameter $\delta = Max[\phi'(x,y) - \phi(x,y)]$ where ϕ' is the next estimate of the solution and ϕ is the estimate from the current iteration and the maximum is taken over the entire array. In each case implement an over/underrelaxation parameter ω so that an $1+\omega = 1$ is standard relaxation, $0 < 1+\omega < 1$ is under relaxation and $1+\omega > 1$ is over relaxation. Note that Jacobi is stable only for $0 < 1+\omega \leq 1$. You're problems may take an input source array $\rho(x,y)$ as well as require boundary conditions $\phi = \phi_{bc}$ on some fixed surface (or set of surfaces). Note that in 2D projection a boundary surface is a curve.

1. Capacitor in a Capacitor
A smaller capacitor of half the size of a larger capacitor is placed between conducting plates but rotated by ninety degrees with the boundary voltages as indicated.



a) Setting $\delta = 10^{-5}$ and for a grid with 20x20 interior points (22x22 including the box boundary points) solve this boundary value problem using the Jacobi, Gauss-Seidel methods (all with $\omega = 0$) How many iterations did it take to reach your tolerance in each case? Plot you're output from the Jacobi routine as a density plot (here and for all density plots be sure to include a colour bar legend), and plot the vertical and horizontal 1D cross sections of ϕ half way through the box. Which method performs better and why?

b) Setting $\delta = 10^{-4}$ and for a grid with 50x50 interior points (52x52 including the box boundary points) find the number of iterations N_{iter} required to reach your tolerance for the values $\omega = (0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99)$ for the Gauss-Seidel and Red-Black methods. Run a finer grid of values of ω near those you find minimize N_{iter} and estimate the optimal value of ω_{opt} in these cases. Make a plot of N_{tier} vs. ω that shows both methods and indicate the value of ω_{opt} on your figure. Plot the output from *either* the Gauss-Seidel or Red-Black routine as a density plot and the vertical and horizontal 1D cross sections of ϕ half way through the box, and compare to you're results from part a). What is the algorithmic advantage of the Red-Black method over the Gauss-Seidel, if any, when considering modern computational hardware?

Quickly, repeat these tests for either the Gauss-Seidel or Red-Black method on a courser grid with 20x20 interior points (like you used in part a)), and find ω_{opt} in this case. Is the value higher or lower than for the 50x50 grid? (It is not necessary to plot this case but you can if you like).

c) Setting $\delta = 10^{-5}$ and using whatever method and ω you like find the solution on a grids with 20x20,60x60, and 200x200 interior points. Plot these side-by-side and comment on convergence. For the 200x200 case only show the 1D vertical and horizontal cross sections 1/3 and 2/3 of the way across the box.

2. Annular Dipole

Setting $\delta = 10^{-4}$, a grid with 120x120 interior points, and using a method of your choice solve for the potential in the presence of a charged cylindrical annulus with an inner diameter one quarter the size of the box and an outer diameter one half the size of the box. The top half has has a uniform density $\rho = 1$ while the bottom half has a uniform negative density $\rho = -1$. Plot both $\rho(x,y)$ and $\phi(x,y)$ as density plots. Describe the shape of $\phi(x,y)$ compared to the shape of $\rho(x,y)$. Can you explain this intuitively given what you know about finite difference schemes?