Computational Physics

PHYS 410 2014W

Assignment #3

Due: Friday, October 17 2014 at 6PM

The gravitational 2-body problem in the limit where $m \ll M$ has the equation of motion:

$$m \frac{d^2 \vec{R}}{dt^2} = -\frac{GMm}{R^2} \frac{\vec{R}}{R}$$

where without loss of generality we may choose $Z = 0$ and take $\vec{R}$ to lie in the (X, Y) plane.

a) By choosing a characteristic length L and a characteristic time T show that this E.O.M can be written in the form

$$\frac{d^2 x}{d\tau^2} = -\gamma \frac{x}{r^3} \quad ; \quad \frac{d^2 y}{d\tau^2} = -\gamma \frac{y}{r^3}$$

$$r^2 = x^2 + y^2$$

Where x, y, τ are dimensionless.
What is γ? Take $\gamma = 1$ for the remainder of the assignment.

Assignment #3 cont.

b) Rewrite these two 2^{nd} order equations as a system of four 1^{st} order equations with variables $\vec{r} = (x, y)$ and $\vec{v} = (v_x, v_y)$

c) Write a program to solve these equations using the Euler and 4^{th} order Runge-Kutta methods with fixed step size h.

For initial conditions $\vec{r} = (\frac{1}{4}, 0)$ $\vec{v} = (0, 1)$

* Solve these equations and plot the trajectory of the planetary body.

Use a time interval from $\tau = 0$ to $\tau = 1$

* What are the initial energy and angular momentum of the system? (per unit mass)
* Plot the change in the energy and (magnitude) of the angular momentum as a function of time.

Assignment # 3 cont.

c) cont.

Euler: Try $h = 10^{-2}, 10^{-4}, 10^{-6}$

RK4: Try $h = 10^{-2}, 10^{-3}, 10^{-4}$

What did you observe in your simulation? Give a short description of your findings and try to explain any anomolies in your trajectories or energy/angular momentum difference plots. How long did the calculation take?
(Hint: Track how many times your velocity/force calculation was called)

d) Implement adaptive stepsize control using $\epsilon = \sqrt{\epsilon_x^2 + \epsilon_y^2}$ as your tolarance error parameter, with the RK4 method.
Experiment until you find a δ that visually produces closed orbits. Compare the speed accuracy and stepsize of the calculation to what you found in c)
Plot the trajectory for a single orbit as a series of "snapshot" points and comment on what you observe. ↳ unconnected