## Physics 501 Problem Set 4

Due at the end of class, Wednesday March $12^{th}$ (late assignments will not be accepted).

Before you start this problem set, please download and read carefully Supplement 18-A to Gasiorowicz, available from the Student Companion Site at Wiley's webpage (there is a link on the course webpage). I will be referring to the equations from this supplement throughout, by number, for example, (18A-5).

Note: the operators in this appendix (**A**, **E**, etc...) appear with their time dependence. Therefore, this appendix is written in the Heisenberg picture.

**1. (a)** Supplement 18-A contains an outline of the derivation of equation (18A-14), the Hamiltonian. By carefully filling in all the algebra, write down a complete derivation following this outline.

**(b)** Starting with formula (18A-21), derive (18A-22), proving that a single photon has momentum $\hbar\mathbf{k}$.

**2. (a)** Compute the commutator $[E_i(\mathbf{r},t), B_j(\mathbf{r}',t)]$ (the subscripts denote vector components in 3D, i=1,2,3). Notice that E and B (approximately) play the role of x and p in the Harmonic Oscillator.

**(b)** Using part (a), compute the commutators of $E_i(\mathbf{r},t)$ and $B_i(\mathbf{r},t)$ with the Hamiltonian (18A-2) to find the time dependence of the fields in the Heisenberg picture. Check that the answers agree with Maxwell's equations.

**3.** Let's check whether the momentum operator (18A-21) is indeed the generator of translations.

**(a)** Compute the commutator $[P_i, A_j(\mathbf{r},t)]$.

**(b)** Defining the translation operator (by an amount $\mathbf{a}$) in the usual way

$$T(\mathbf{a}) = e^{-i\mathbf{P}\cdot\mathbf{a}/\hbar}$$

compute $T(\mathbf{a})\mathbf{A}(\mathbf{r},t)T(\mathbf{a})^{-1}$ and show that it has the desired form.

You will need to use the Baker-Hausdorff formula

$$e^X Y e^{-X} = Y + [X,Y] + \frac{1}{2}[X,[X,Y]] + \frac{1}{6}[X,[X,[X,Y]]] + \ldots + \frac{1}{n!}[X,[X,[X\ldots[X,Y]\ldots]]] + \ldots$$