

## Formula Sheet

$$PV = NkT \qquad PV = nRT \qquad kN_A = R$$

$$\bar{K}_{trans} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

$$U_{trans} = N\bar{K}_{trans} = \frac{3}{2}NkT = \frac{3}{2}PV$$

$$U_{thermal} = N \cdot f \cdot \frac{1}{2}kT$$

$$\Delta U = W + Q$$

$$W = -P\Delta V \quad \text{for constant pressure}$$

$$W = -\int P \, dV \quad \text{for variable pressure}$$

For isothermal compression,  $PV = \text{const}$

For adiabatic compression,  $VT^{f/2} = \text{const}$  and  $V^{(f+2)/f}P = \text{const}$

$$C = Q/\Delta T$$

For the ideal gas,  $C_V = fNk/2$  and  $C_P = (f+2)Nk/2$

Enthalpy  $H = U + PV$

Multiplicity of the two-state paramagnet:

$$\Omega(N, q) = \binom{N}{q} = \frac{N!}{q!(N-q)!}$$

Multiplicity of the Einstein solid:

$$\Omega(N, q) = \binom{N+q-1}{q} = \frac{(N+q-1)!}{q!(N-1)!}$$

$$N! \approx N^N e^{-N} \sqrt{2\pi N}$$

$$N! \approx N^N e^{-N}$$

$$\ln N! \approx N \ln N - N$$

For  $x$  small, we have:

$$f(x) = f(0) + \left. \frac{df}{dx} \right|_{x=0} x + \frac{1}{2} \left. \frac{d^2 f}{dx^2} \right|_{x=0} x^2$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{1}{2}\alpha(\alpha-1)x^2 + \dots$$

$$S = k \ln \Omega$$

Multiplicity of a monatomic ideal gas:

$$\Omega_N = \frac{V^N}{N!} \left( \frac{2\pi m U}{h^2} \right)^{3N/2} \frac{1}{\left(\frac{3N}{2}\right)!} = f(N) V^N U^{\frac{3N}{2}}$$

The entropy of a monatomic ideal gas is

$$S(N, V, U) = kN \left[ \ln \left( \frac{V}{N} \right) + \frac{3}{2} \ln \left( \frac{U}{N} \right) + \frac{3}{2} \ln \left( \frac{4\pi m}{3h^2} \right) + \frac{5}{2} \right]$$

$$T \equiv \left( \left( \frac{\partial S(U, N, V)}{\partial U} \right)_{N, V} \right)^{-1}$$

$$P = T \left( \frac{\partial S}{\partial V} \right)_{U, N} \quad \mu = -T \left( \frac{\partial S}{\partial N} \right)_{U, V} = \left( \frac{\partial U}{\partial N} \right)_{S, V}$$

$$dS = \frac{Q}{T} \quad \text{quasistatic, reversible only}$$

$$\Delta S = \int_{T_i}^{T_f} \frac{C_{[V \text{ or } P]}}{T} dT$$

$$dU = T dS - P dV + \mu dN$$

Efficiencies and Costs of Operation:

$$e = \frac{W}{Q_{in}}, \text{ where } W = Q_{in} - Q_{out}, \quad Q_{in} = Q_h, \quad Q_{out} = Q_c$$

$$e_{max} = e_{Carnot} = 1 - \frac{T_C}{T_H}$$

$$\text{COP} = \frac{Q_{in}}{W}, \text{ where } W + Q_{in} = Q_{out}, \quad Q_{in} = Q_c, \quad Q_{out} = Q_h$$

$$\text{COP}_{max} = \frac{T_c}{T_h - T_c}$$