Formula Sheet

$$PV = NkT \qquad PV = nRT \qquad kN_A = R$$

$$\bar{K}_{trans} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

$$U_{trans} = N\bar{K}_{trans} = \frac{3}{2}NkT = \frac{3}{2}PV$$

$$U_{thermal} = N \cdot f \cdot \frac{1}{2}kT$$

$$\Delta U = W + Q$$

$$W = -P\Delta V \quad \text{for constant pressure}$$

$$W = -\int P\ dV \quad \text{for variable pressure}$$

For isothermal compression, $PV =$ const

For adabiatic compression, $VT^{f/2} =$ const and $V^{(f+2)/f}P =$ const

$$C = Q/\Delta T$$

For the ideal gas, $C_V = fNk/2$ and $C_P = (f+2)Nk/2$

Multiplicity of the two-state paramagnet:

$$\Omega(N,q) = \binom{N}{q}$$

Multiplicity of the Einstein solid:

$$\Omega(N,q) = \binom{N+q-1}{q}$$

$$\begin{aligned} N! &\approx N^N e^{-N}\sqrt{2\pi N} \\ N! &\approx N^N e^{-N} \\ \ln N! &\approx N\ln N - N \end{aligned}$$

For x small, we have:

$$f(x) = f(0) + \frac{df}{dx}|_{x=0}\ x \ + \ \frac{1}{2}\ \frac{d^2f}{dx^2}|_{x=0}\ x^2$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{1}{2}\alpha(\alpha-1)x^2 + \ldots$$

$$S = k\ln\Omega$$

$$T \equiv \left(\left(\frac{\partial S(U,N,V)}{\partial U} \right)_{N,V} \right)^{-1}$$

$$P = T \left(\frac{\partial S}{\partial V} \right)_{U,N} \qquad \mu = -T \left(\frac{\partial S}{\partial N} \right)_{U,V} = \left(\frac{\partial U}{\partial N} \right)_{S,V}$$

$$dS = \frac{Q}{T} \qquad \text{quasistatic, reversible only}$$

$$\Delta S = \int_{T_i}^{T_f} \frac{C_{[V \text{ or } P]}}{T} \, dT$$

The entropy of a monatomic ideal gas is:

$$S(N,V,U) = kN \left[\ln\left(\frac{V}{N}\right) + \frac{3}{2} \ln\left(\frac{U}{N}\right) + \frac{3}{2} \ln\left(\frac{4\pi m}{3h^2}\right) + \frac{5}{2} \right]$$

Efficiencies and Costs of Operation:

$$e = \frac{W}{Q_{in}}, \text{ where } W = Q_{in} - Q_{out},\ Q_{in} = Q_h,\ Q_{out} = Q_c$$

$$e_{max} = e_{Carnot} = 1 - \frac{T_C}{T_H}$$

$$\text{COP} = \frac{Q_{in}}{W}, \text{ where } W + Q_{in} = Q_{out},\ Q_{in} = Q_c,\ Q_{out} = Q_h$$

$$\text{COP}_{max} = \frac{T_c}{T_h - T_c}$$

Enthalpy $H = U + PV$
Helmholtz free energy $F = U - ST$
Gibbs free energy $G = U + PV - ST = H - ST$

$$\begin{aligned} dU &= T\,dS - P\,dV + \mu dN, && \text{U=U(S,V,N)} \\ dH &= T\,dS + V\,dP + \mu dN, && \text{H=H(S,P,N)} \\ dF &= -S\,dT - P\,dV + \mu dN, && \text{F=F(T,V,N)} \\ dG &= -S\,dT + V\,dP + \mu dN, && \text{G=G(T,P,N)} \end{aligned}$$

$$G = N\mu$$

Clausius-Clapeyron:

$$\frac{\partial P}{\partial T} = \frac{L}{T\Delta V}$$

At constant V, U and N, S goes to a maximum
At constant V, T and N, F goes to a minimum
At constant P, T and N, G goes to a minimum

$$\beta \equiv \frac{1}{kT}$$

s - state label

$$Z(T) \equiv \sum_{\text{all states s}} e^{-\beta E(s)}$$

$$Z(T) = \sum_{\text{all energies E}} \Omega(E)\ e^{-\beta E}$$

$$P(s) = \frac{1}{Z}\ e^{-\beta E(s)}$$

$$P(E) = \frac{1}{Z}\ \Omega(E)\ e^{-\beta E}$$

$$\overline{X} = \sum_s X(s)P(s)$$

$$U = N\overline{E} = -\frac{N}{Z_1}\frac{d}{d\beta}Z_1(\beta) = -\frac{1}{Z_N}\frac{d}{d\beta}Z_N(\beta)$$

Maxwell speed distribution $\mathcal{D}(v)$ in an ideal gas:

$$\mathcal{D}(v) \ = \ \left(\frac{m}{2\pi kT}\right)^{3/2}\ 4\pi v^2\ e^{-mv^2/2kT}$$

$$F = -kT\ln Z$$

$$S = -\frac{\partial F}{\partial T}|_{V,N} = k\ln Z + \frac{U}{T}$$

$$Z_N = (Z_1)^N \quad \text{for distinquishable particles}$$

$$Z_N = \frac{1}{N!}(Z_1)^N \quad \text{for indistinquishable particles}$$

Entropy of the ideal gas with f quadratic degs of freedom

$$S(T,V,N) = kN\left[\ln\left((\text{constant})\frac{V}{N}\,(kT)^{f/2}\right) + \frac{f+2}{2}\right]$$

Bose-Einstein (bosons) statistics:

$$\overline{n}_{BE} = \frac{1}{e^{\beta(\epsilon-\mu)} - 1}$$

Fermi-Dirac (fermions) statistics:

$$\overline{n}_{FD} = \frac{1}{e^{\beta(\epsilon-\mu)} + 1}$$

Planck distribution (blackbody radiation):

$$\mathcal{I}(\epsilon) = u(\epsilon) = \frac{8\pi\left(\frac{\epsilon}{hc}\right)^3}{e^{\beta\epsilon} - 1}$$

$$\frac{U}{V} = \int_0^\infty \mathcal{I}(\epsilon)d\epsilon = \int_0^\infty u(\epsilon)d\epsilon = \frac{8\pi^5}{15}\frac{(kT)^4}{(hc)^3} \ \sim\ T^4$$

Stefan's law (blackbody emission):

$$\frac{\text{power}}{A} = \frac{c}{4}\frac{U}{V} = \sigma T^4$$

$$\sigma = \frac{2\pi^5}{15}\frac{k^4}{(hc)^3} = 5.67\cdot 10^{-8}\frac{W}{m^2K^4}$$