Physics 313 Problem Set 6

Important concepts from lectures 15-18

Note on signs: In this chapter (chapter 5) and in this chapter only, the sign convention is that all quantities (Q and W) are positive, and we explicitly say whether they are 'in' or 'out'. This means that sometimes $\Delta U = Q - W$, or $\Delta U = -Q + W$ or whatever. If you find it confusing, feel free to keep the same convention as we had before, as long as you warn the marker, and keep it consistent.

Another note: I do not expect you to memorize all the cycles! You **should** be able to draw the Carnot cycle from memory (it is rather simple and very important). The others (Stirling, Otto, Diesel, Rankine...) you should just understand.

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A heat engine is anything which:

- Runs in a closed cycle (returns back to its starting position at every cycle)
- Takes in heat Q_{in} from a hot reservoir (on every cycle)
- Expels waste heat Q_{out} into a cold reservoir (on every cycle)
- Does useful work W (on every cycle)
- has a T_H and a T_C: the highest and the lowest temperature the engine is in contact with

The efficiency of the heat engine is defined to be

$$e = \frac{W}{Q_{in}}, \text{ where } W = Q_{in} - Q_{out}$$

The maximum possible efficiency allowed by the second law is

$$e_{max} = 1 - \frac{T_C}{T_H}$$

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A refrigerator is a heat engine running in reverse. Its Coefficient Of Performance (COP) is the amount of heat removed from the cold reservoir divided by the work done, and is bounded from above by the second law:

$$COP = \frac{Q_{removed}}{W} \leq \frac{T_c}{T_h - T_c}$$

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The Carnot cycle is made up of two isothermal curves at temperatures T_H and T_C and two adiabatic curves connecting them. The efficiency of the Carnot cycle is e_{max}, as long as the cycle is completed reversibly and quasi statically. We proved this in class three different ways - it is important.

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There are two idealized cycles for car and truck engines: the Otto cycle and the Diesel cycle. Neither is as efficient as a Carnot engine.

The Otto cycle compresses the gasoline and air mixture, which is then ignited by a spark from the spark plug. In the Diesel engine, the air is compressed first, to a much higher pressure and temperature, the gasoline is added after the compression, and it self-ignites due to the high temperature. The higher burning temperature of the Diesel engine allows for a higher theoretical efficiency.

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The Stirling engine is a clever set up which can theoretically have the same efficiency as a Carnot engine.

Just like the Carnot cycle, the Stirling cycle has two isothermal curves at T_H and T_C. Connecting them, however, are two constant volume stages. The clever bit is that the heat (and the associated entropy) given off while the pressure is lowered at constant volume is stored in the recycler, and transfered back to the gas while raising the pressure at constant volume in the second half of the cycle. Thus, no extra heat is given up to the outside, no entropy is generated, and the cycle has efficiency e_{max}.

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The steam engine is how power is produced commercially. It runs on a Rankine cycle. In this cycle, water is pressurized and boiled to turn it into very hot steam, the steam does work by expanding adiabatically, and finally is cooled and condensed back to water.

Since water cannot be treated as an ideal gas, efficiencies are computed from enthalpy tables and entropy tables.

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Enthalpy. Definition: $H = U + PV$

If a process happens at constant pressure, the added heat is equal to the change in enthalpy $\Delta H = Q$.

Problem Set

Due at the end of class, Wednesday October 22^{th} (late assignments will not be accepted).

1. Explain carefuly how, if you were given a refrigerator whose COP is higher than the ideal, $\frac{T_c}{T_h - T_c}$ you could violate the second law of thermodynamics.

2. Schroeder Problem 4.3, page 124.

3. Let's assume that at its hottest, my car burns fuel at 1500°C (a reasonable guess), and that the cold air comes in at 15°C.

(a) What is the ideal efficiency of a Carnot cycle operating between these two temperatures?

(b) Let's say that the engine in my car has a (typical) operating efficiency of 25%. My car burns 7.5 L of gasoline per 100km. How much would it burn if the engine had the ideal efficiency from part (a)?

(c) With 25% efficiency, my car is wasting 75% of the energy available in the gasoline. Where is all this energy going?

4. Schroeder Problem 4.10, page 129.

5. Schroeder Problem 4.13, page 129.

6. **(a)** Assuming that the Otto cycle consist of two constant volume segments at volumes V_1 and V_2 ($V_1 > V_2$) and two adiabatic segments connecting them (as shown in the book in figure 4.5), and that the working substance is an ideal gas, show that the efficiency of the Otto cycle is

$$e = 1 - \left(\frac{V_2}{V_1}\right)^{2/f}$$

(b) Show that the efficiency in part (a) can be written as

$$e = 1 - \frac{T_1}{T_2}$$

7. Schroeder, Problem 4.21 (b-c) only, page 133-134. We did part (a) in class.

Extra practice (not for credit):

- If you feel you need extra practice computing efficiency of cycles, try Schroeder, Problem 4.20, page 133. Note: the Diesel engine is less efficient at any given compression ratio than the Otto engine, but it operates at much higher compression ratios than any Otto engine could.
- Schroeder, Problem 4.12, page 129, is a nice little puzzle.