Physics 313 Problem Set 5

Important concepts from lectures 11-14

From considerations of mechanical equilibrium, we obtained the following definition of pressure

$$P = T\left(\frac{\partial S}{\partial V}\right)_{U,N}$$

—

$W = -PdV$ and $Q = SdT$, $dS = \frac{Q}{T}$ for all reversible, quasi-static processes.
Otherwise: $dS > \frac{Q}{T}$

—

Chemical potential μ: the quantity which is the same for 2 systems in diffusive equilibrium.

$$\mu = -T\left(\frac{\partial S}{\partial N}\right)_{U,V}$$

Particles tend to flow from systems with higher μ to those with lower μ.

—

The thermodynamic identity $\boxed{dU = TdS - PdV + \mu dN}$

—

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V}$$

—

If it takes energy to get a particle into the system, this energy will be reflected in the chemical potential.

—

Ideal, two-state paramagnet:

The total energy for a state with $N_\uparrow$ $\uparrow$s and $N_\downarrow$ $\downarrow$s is $U = (N_\downarrow - N_\uparrow)B\mu$ and the magnetization is $M = (N_\uparrow - N_\downarrow)\mu$ so that $U = -\mu M$.

In class, you saw a derivation of the following:

$$U = -N\mu B \tanh\left(\frac{\mu B}{kT}\right)$$

$$M = N\mu \tanh\left(\frac{\mu B}{kT}\right)$$

Notice that negative magnetization = negative temperature =
= system ever hotter than ($T = +\infty$).
For T large and positive,

$$M = N\left(\frac{\mu^2 B}{kT}\right) \sim \frac{1}{T}$$

This is known as Curie's Law: magnetization is inversely proportional to temperature.

Problem Set

Due at the end of class, Wednesday October 15^{th} (late assignments will not be accepted).

1. Starting with the thermodynamic identity for dU, show that

$$P = -\left(\frac{\partial U}{\partial V}\right)_{S,N}$$

What is the physical interpretation of this formula?

2. Schroeder Problem 3.31, page 114.

3. Sketch (or use a computer to plot) a graph of the entropy of the two state paramagnet as a function of temperature. Don't forget to explain in detail how you obtained the plot! (You may use any formulas/figures derived/shown in section 3.3 of the book or in class.)

4. **Part 1** Schroeder Problem 1.16, page 8.
Part 2 Schroeder Problem 3.37, page 119-120.
[This question is especially important for atmospheric science students.]

Extra (not for credit): If you are looking for some extra problems, Schroeder 3.34 is a fun one, and it teaches you about the connection between thermodynamics and polymers.