

Physics 313 Problem Set 2

Important concepts from lectures 2-4

Notation:

T - temperature

P - pressure

V - volume

n - number of moles

N - number of molecules/atoms

U - internal energy

W - work

Q - heat

f - number of quadratic degrees of freedom

C - heat capacity

C_V - heat capacity at constant volume

C_P - heat capacity at constant pressure

R - gas constant

k - Boltzmann constant

N_A - Avogadro's number

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When two objects at different temperatures are in thermal contact with each other, energy (in form of heat) flows spontaneously from the hotter object to a colder one. Once the objects have been in contact long enough, they reach equilibrium and energy stops flowing.

When two objects are in equilibrium, they have the same temperature. So, we define: temperature is "this thing" which is the same for all objects in equilibrium. Temperature is also (another definition) a measure of the tendency of an object to give off heat.

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Every system has certain properties which depend only on its current physical state (and not on how the system was prepared). These are called functions of state. Examples: temperature, pressure, volume, density, number of molecules, internal energy...

Heat is **not** a function of state, and neither is work!

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To know everything about an ideal gas, all you need is 3 parameters (N, T and V; or P, V and N; or ...) There are however 4 basic parameters of a gas: P, V, N, T. They cannot be independent, then. The equation which links them is the Ideal Gas Law:

$$\boxed{PV = NkT} \quad \text{or} \quad PV = nRT$$

The constants k and R are connected by Avogadro's number: $k N_A = R$ (to remember this, think 'k is per molecule, R is per mole').

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Average translational kinetic energy of a molecule in ideal gas is

$$\bar{K}_{trans} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

The total internal energy due to translational motion alone is

$$U_{trans} = N\bar{K}_{trans} = \frac{3}{2}NkT = \frac{3}{2}PV$$

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The Equipartition Theorem focuses on quadratic degrees of freedom. These have energies of the form

$$\frac{1}{2}(\text{const})(\text{coordinate})^2$$

where the coordinate is in phase space (position + momentum). Examples:

$$\frac{1}{2}m(v_x)^2 \quad \text{translational kinetic energy}$$

$$\frac{1}{2}k(x)^2 \quad \text{vibrational potential energy}$$

$$\frac{1}{2}I(\omega)^2 \quad \text{rotational kinetic energy}$$

Examples of how to count the available degrees of freedom were given in class. Remember that at any given temperature some degrees of freedom might be frozen out.

Equipartition Theorem:

$$U_{thermal} = N \cdot f \cdot \frac{1}{2}kT$$

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Temperature - tendency of the system to give up heat spontaneously.

Energy (U) - total energy in the system. A sum of all kinetic and potential energies of all particles, all internal energies, quantum energies, etc...

Work (W) - energy transfer via macroscopic variables (ex, pushing on a piston, changing the magnetic field)

Heat (Q) - energy transferred on a molecular level due to a difference in temperatures.

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Types of heat exchange: conduction, convection, radiation.

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First law of thermodynamics (also known as Conservation of Energy):

$$\Delta U = W + Q$$

Q and W are defined as changes only, you cannot say 'the system has this much heat and this much work in it', you can only say 'this much work was done on the system' and 'this much heat has flown into it'.

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Compression work

$$W = -P\Delta V \quad \text{for constant pressure}$$

$$W = - \int P \, dV \quad \text{for variable pressure}$$

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We discussed in class the compression of ideal gas: adiabatic (no heat flow) and isothermal (constant temperature).

- for isothermal compression, $PV = \text{const}$
- for adiabatic compression, $VT^{f/2} = \text{const}$ and $V^{(f+2)/f}P = \text{const}$

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Heat capacity is the amount of heat you need to add to a system to raise its temperature by a unit. $C = Q/\Delta T$.

This definition needs to be supplemented with information about the conditions under which the heating is happening, such as constant volume or constant pressure. For example, for the ideal gas, $C_V = fNk/2$ and $C_P = (f+2)Nk/2$.

Enthalpy is defined as $H = U + PV$.

Problem Set

Due at the end of class, Wednesday September 17th (late assignments will not be accepted).

1. Compute the root-mean-squared velocity (ie, $(\overline{v^2})^{1/2}$) of oxygen, nitrogen, carbon dioxide, helium and xenon molecules/atoms at room temperature, assuming that the equipartition of energy holds.
2. How many quadratic degrees of freedom are there per molecule in the following cases:
 - (a) Cubic crystal in D spatial dimensions
 - (b) Monoatomic gas in D spatial dimensions
 - (c) Diatomic gas in 2 spatial dimensions, assuming that
 - (i) Vibrations are frozen out, but rotations are not
 - (ii) Nothing is frozen out
3. The table below contains constant volume heat capacities (taken from the CRC handbook for Physics and Chemistry), for a few substances at room temperature. Estimate the constant volume heat capacities of these substances using the equipartition theorem, and compare to the data in the table. Is the equipartition theorem doing well?

Substance	Phase at room temperature	C_v [J/(g K)]
Fluorine (F_2)	gas	0.605
Hydrogen (H_2)	gas	10.1
Helium	gas	3.11
Ammonia (NH_3)	gas	1.61
Iron	solid	0.444
Aluminum	solid	0.900

4. Start with one mole of pure oxygen O_2 at temperature T and volume V .
 - (a) The gas is compressed adiabatically until its pressure is tripled. Compute (as a function of V and T), the final volume at the end of the compression, the final temperature and the total work done.
 - (b) After the adiabatic compression in part (a), the gas is allowed to cool back down to its original temperature T at constant volume. Compute the final pressure at the end of this cooling, and the amount of heat emitted.
5. Schroeder problem 1.37 (page 26)
6. Schroeder problem 1.47 (page 33). You will find some useful data just above the problem.