

Physics 203 Quantum Mechanics facts

h is the Planck's constant, $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$

1. Quantum Harmonic Oscillator

For an oscillator with a classical oscillation frequency f , the energy spectrum is $(hf)n + hf/2$ for $n \geq 0$ an integer. The spacing between the energy levels is hf .

Thermal excitation of the vibration occurs only when $kT \gg hf$. When $kT < hf$, the vibration is said to be frozen out and does not contribute to the number of degrees of freedom. Typically, at room temperature, vibrations in small molecules (H_2O , N_2 , NH_3 , etc...) **are** frozen out. For example, for nitrogen, the period of oscillation is 12fs, and $hf = 0.34\text{eV}$, which is larger than $kT = (1/40)\text{eV}$ for $T = 300\text{K}$ – the vibrations are definitely frozen out.

2. Uncertainty relation

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} = \frac{h}{4\pi}$$

In the $x - p$ plane, we should think of every particle as occupying a small tile of area proportional to $(\Delta x)(\Delta p) \sim h$. (h^{-1} is a large number, so we can drop any factor of 4 and π .)

3. Two state paramagnet

Each site contains a spin which has only two states - \uparrow and \downarrow . The energy for these two states is $\pm B\mu$. The total energy for a state with N_\uparrow \uparrow s and N_\downarrow \downarrow s is $U = (N_\downarrow - N_\uparrow)B\mu$ and the magnetization is $M = (N_\uparrow - N_\downarrow)\mu$ so that $U = -\mu M$. The multiplicity is

$$\Omega = \binom{N_\downarrow + N_\uparrow}{N_\uparrow} = \binom{N}{q}$$

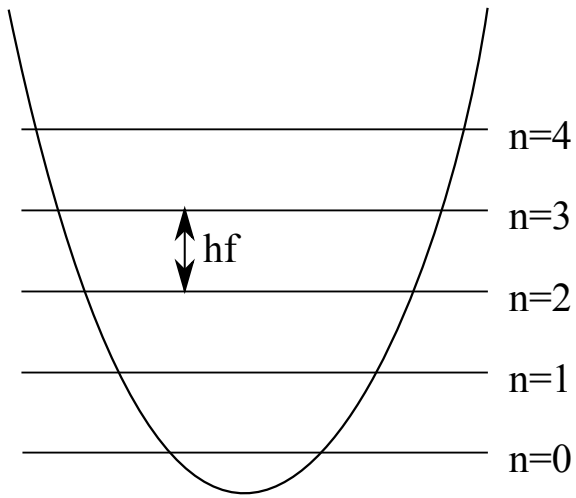
where N is the total number of sites and q is the number of energy quanta in the system (i.e. the number of spins flipped to the state of higher energy).

4. Einstein Solid

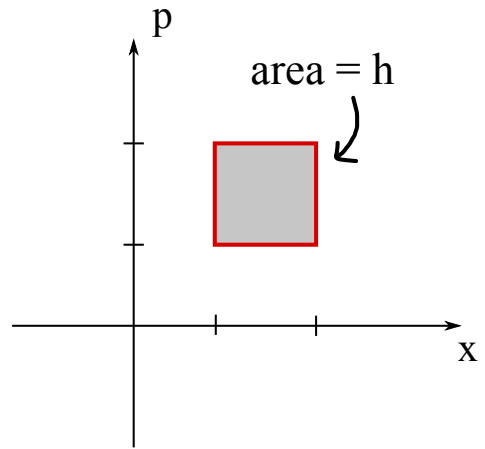
This is a collection of N QHO (see above). Every QHO has some non-negative number of energy quanta n . If we denote by q the total number of energy quanta in the system, the total internal energy is $U = q(hf)$ and the multiplicity is

$$\Omega = \binom{N + q - 1}{q}$$

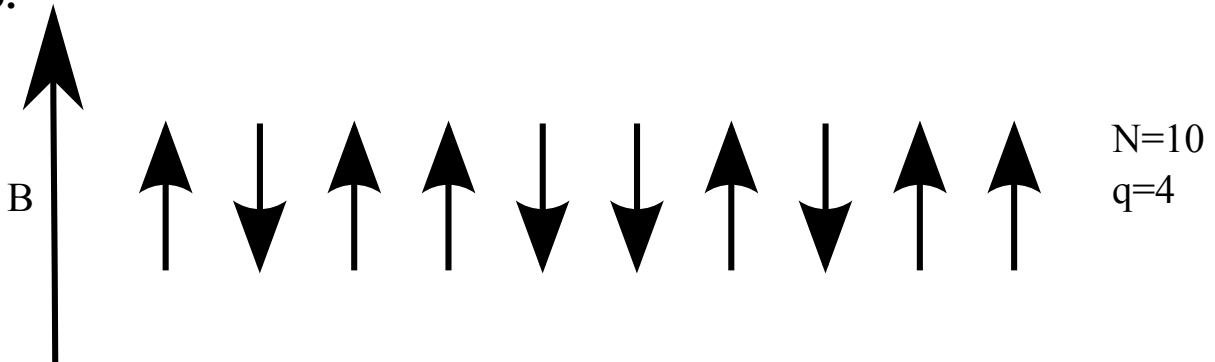
1.



2.



3.



4.

