here for only a brief introduction, we shall discuss relativistic kinematics in some detail and merely outline relativistic dynamics, omitting derivations. We shall begin by looking at a historically important experiment related to the theory of special relativity, the Michelson-Morley experiment.

1-1 The Michelson-Morley Experiment

All waves except electromagnetic waves require a medium for their propagation. The speed of a wave depends on the properties of the medium. For sound waves, for example, the medium is air, and absolute motion, i.e., motion relative to the still air, can be detected. The Doppler effect for sound depends not only on the relative motion of the source and listener but on the absolute motion of each relative to the air. It was natural to expect that some kind of medium exists which supports the propagation of light and other electromagnetic waves. Such a medium, called the *ether,* was proposed in the nineteenth century. The ether as proposed would have had to possess unusual properties. Although it would require great rigidity to support waves of such high velocity (recall that the velocity of waves on a string depends on the tension of the string), it must introduce no drag force on the planets, as their motion is fully accounted for by the law of gravitation.

It was of considerable interest to determine the velocity of the earth relative to the ether. Maxwell pointed out that in measurements of the speed of light, the earth's speed v relative to the ether appears only in the second order v^2/c^2, an effect then considered too small to measure. Such measurements determine the time for a light pulse to travel to and from a mirror. Figure 1-1 shows a light source and a mirror a distance L apart. If we assume that both are moving with speed v through the ether, classical theory predicts that the light will travel toward the mirror with speed $c - v$ and back with speed $c + v$ (both speeds relative to the mirror and light source). The time for the total trip will be

$$t_1 = \frac{L}{c - v} + \frac{L}{c + v} = \frac{2cL}{c^2 - v^2} = \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1} \qquad \text{1-1}$$

For v much smaller than c, we can expand this result using the binomial expansion

$$(1 + x)^n \approx 1 + nx + \cdots \qquad \text{for } x \ll 1$$

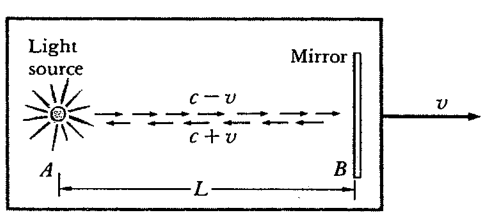


Figure 1-1
Light source and mirror moving with speed v relative to the "ether." According to classical theory, the speed of light relative to the source and mirror would be $c - v$ toward the mirror and $c + v$ away from the mirror.

Then

$$t_1 \approx \frac{2L}{c}\left(1 + \frac{v^2}{c^2} + \cdots\right) \tag{1-2}$$

If we take the orbital speed of the earth about the sun as an estimate of v, we have $v \approx 3 \times 10^4$ m/sec $= 10^{-4}c$ and $v^2/c^2 = 10^{-8}$. Thus the correction for the earth's motion is small indeed.

Albert A. Michelson realized that, although this effect is too small to be measured directly, it should be possible to determine v^2/c^2 by a difference measurement. Figure 1-2a is a diagram of his apparatus, called a *Michelson interferometer.* Light from the source is partially reflected and partially transmitted by mirror A. The transmitted beam travels to mirror B and is reflected back to A. The reflected beam travels to mirror C and is reflected back to A. The two beams recombine and form an interference pattern, which is viewed by an observer at O. Equation 1-2 gives the classical result for the round-trip time t for the transmitted beam. Since the reflected beam travels (relative to the earth) perpendicular to the earth's velocity, the velocity of this beam relative to earth (according to classical theory) is the vector difference $\mathbf{u} = \mathbf{c} - \mathbf{v}$. The magnitude of $\mathbf{u}$ is $\sqrt{c^2 - v^2}$; so the round-trip time for this beam is

$$\begin{aligned} t_2 &= \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \\ &\approx \frac{2L}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots\right) \end{aligned} \tag{1-3}$$

where again the binomial expansion has been used. There is thus a time difference:

$$\begin{aligned} \Delta t = t_1 - t_2 &\approx \frac{2L}{c}\left(1 + \frac{v^2}{c^2}\right) - \frac{2L}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) \\ &\approx \frac{Lv^2}{c^3} \end{aligned} \tag{1-4}$$

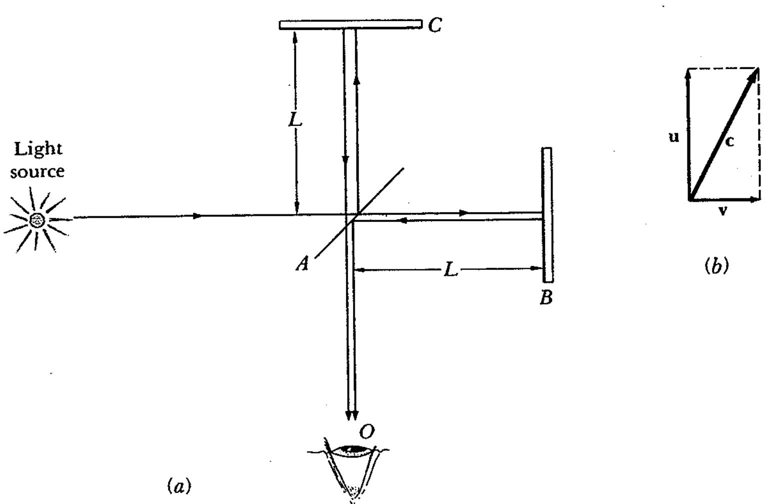


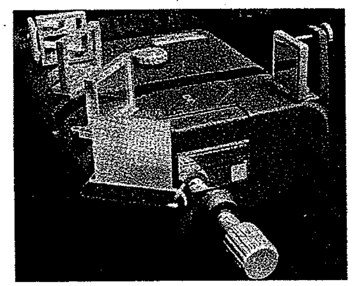
Figure 1-2
(a) Schematic drawing of the Michelson interferometer. (b) According to classical theory, if the interferometer moves to the right with velocity $\mathbf{v}$ relative to the ether, the light must move with velocity $\mathbf{c}$ in the direction shown (relative to the ether) to strike the upper mirror. Its velocity relative to the interferometer is then $\mathbf{u} = \mathbf{c} - \mathbf{v}$, and its speed is $u = \sqrt{c^2 - v^2}$.

The time difference is to be detected by observing the interference of the two beams of light. Because of the difficulty of making the two paths of equal length to the precision required, the interference pattern of the two beams is observed and then the whole apparatus rotated 90°. The rotation produces a time difference given by Equation 1-4 for each beam. The total time difference of $2\,\Delta t$ is equivalent to a path difference of $2c\,\Delta t$. The interference fringes observed in the first orientation should thus shift when the apparatus is rotated by a number of fringes ΔN, given by

$$\Delta N = \frac{2c\,\Delta t}{\lambda} = \frac{2L}{\lambda}\frac{v^2}{c^2} \qquad \text{1-5}$$

where λ is the wavelength of the light. In Michelson's first attempt, in 1881, L was about 1.2 m and λ was 590 nm. For $v^2/c^2 = 10^{-8}$, ΔN was expected to be 0.04 fringe.

When no shift was observed, Michelson reported the null result even though the experimental uncertainties were estimated to be about the same order of magnitude as the expected effect. In 1887, when he repeated the experiment with Edward W. Morley, he used an improved system for rotating the apparatus without introducing a fringe shift because of mechanical strains, and he increased the effective path length L to about 11 m by a series of multiple reflections. Figure 1-3 shows the configuration of the Michelson-Morley apparatus. For this attempt, ΔN was expected to be about 0.4 fringe, about 20 to 40 times the minimum shift observable. Once again, no shift was observed. The experiment has since been repeated under various conditions by a number of people, and no shift has ever been found.



A student-type Michelson interferometer. The fringes are produced on a ground-glass screen by light from a laser. (*Courtesy of Libor Velinsky.*)

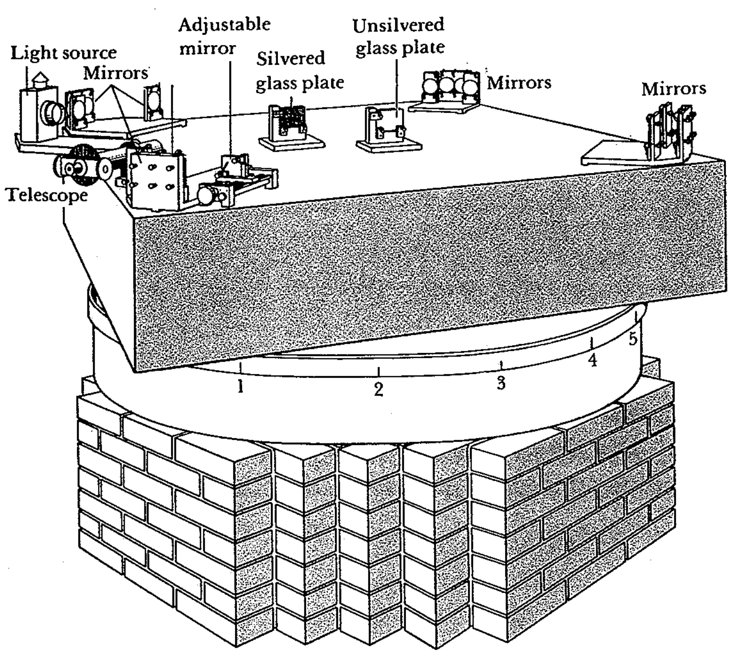


Figure 1-3
Drawing of Michelson-Morley apparatus used in their 1887 experiment. The optical parts were mounted on a sandstone slab 5 ft square, which was floated in mercury, thereby reducing the strains and vibrations that had affected the earlier experiments. Observations could be made in all directions by rotating the apparatus in the horizontal plane. (*From R. S. Shankland, "The Michelson-Morley Experiment," Copyright © November 1964 by Scientific American, Inc. All rights reserved.*)

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Albert A. Michelson in his laboratory. (*Courtesy of the Niels Bohr Library, American Institute of Physics.*)

The null result of the Michelson-Morley experiment is easily understood in terms of the Einstein postulates. According to postulate 1, absolute uniform motion cannot be detected. We can consider the whole apparatus and the earth to be at rest. No fringe shift is expected when the apparatus is rotated 90° since all directions are equivalent. It should be pointed out that Einstein did not set out to explain this experiment. His theory arose from his considerations of the theory of electricity and magnetism and the unusual property of electromagnetic waves—namely, that they propagate in a vacuum. In his first paper, which contains the complete theory of special relativity, he made only a passing reference to the Michelson-Morley experiment, and in later years he could not recall whether he was aware of the details of the experiment before he published his theory.

1-2 Consequences of Einstein's Postulates

An immediate consequence of the two Einstein postulates is that

Every observer obtains the same value for the speed of light independent of the relative motion of sources and observers.

Consider a light source S and two observers, R_1 at rest relative to S and R_2 moving toward S with speed v, as shown in Figure 1-4a. The speed of light measured by R_1 is $c = 3 \times 10^8$ m/sec. What is the speed measured by R_2? The answer is not $c + v$. By postulate 1, Figure 1-4a is equivalent to Figure 1-4b, in which R_2 is pictured at rest and the source S and R_1 are moving with speed v. That is, since absolute motion has no meaning, it is not possible to say which is really moving and which is at rest. By postulate 2, the speed of light from a moving source is inde-