Addition of velocities

-> clicker question

A spaceship has a gun which can fire bullets forward with very high velocity u.
The spaceship is moving with velocity v w.r.t. a planet.
How fast are the bullets going?

SHIP'S FRAME:

$x=0,\ t=0,$ BANG!
EVENT A

$t=T$
$x=uT$
EVENT B

PLANET'S FRAME

L.T.:
$$\begin{cases} \tilde{x} = \gamma\,(x + vt) \\ \tilde{t} = \gamma\left(t + \frac{v}{c^2}x\right) \end{cases}$$

$$x_A = 0 \qquad t_A = 0$$
$$\tilde{x}_A = 0 \qquad \tilde{t}_A = 0$$

$$x_B = uT \qquad t_B = T$$
$$\tilde{x}_B = \gamma\,(uT + vT) = \gamma T\,(u+v)$$
$$\tilde{t}_B = \gamma\left(T + \frac{v}{c^2}uT\right) = \gamma T\left(1 + \frac{uv}{c^2}\right)$$

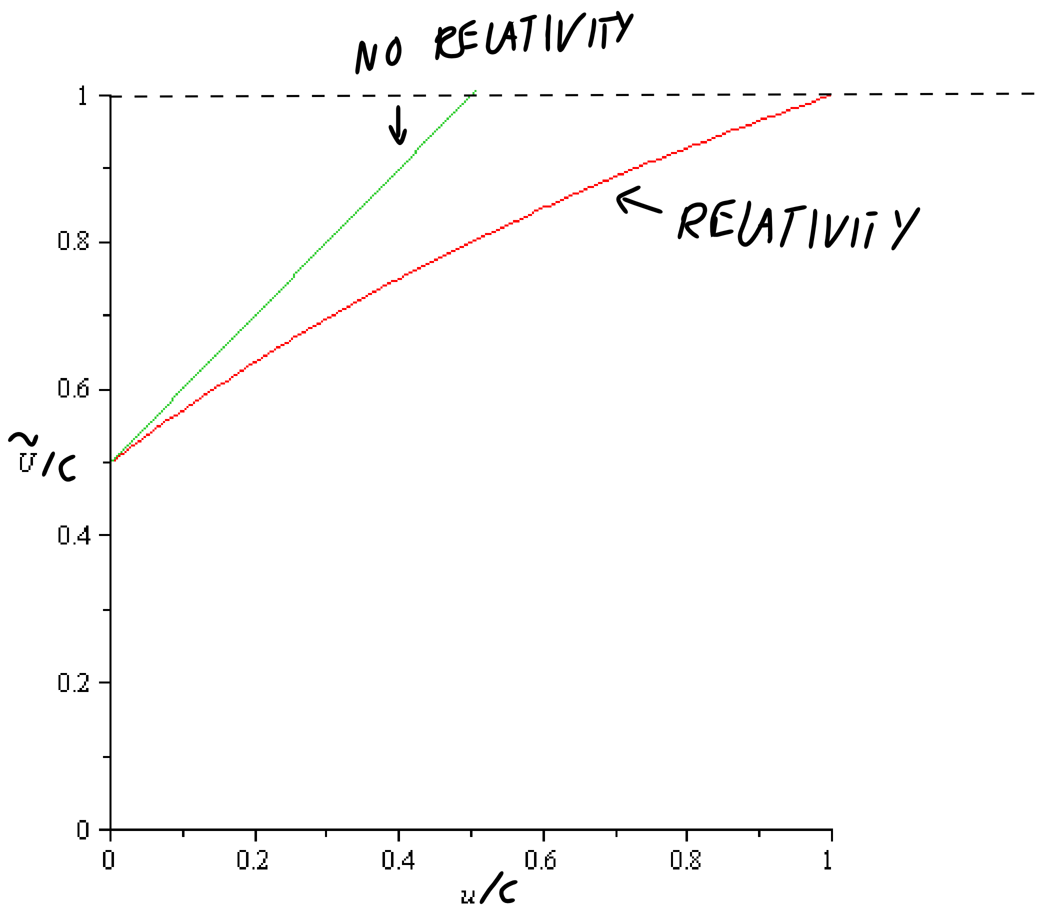
How fast is the bullet going?

$$\tilde{u} = \frac{\Delta x}{\Delta t} = \frac{\gamma T(u+v)}{\gamma T\left(1+\frac{uv}{c^2}\right)} = \frac{u+v}{1+\frac{uv}{c^2}}$$

Compare with just u+v
the uv/c^2 term is the relativistic correction
Because of it, the speed never goes over c

-> clicker question

Plot: let v=0.5c. Plotting u w.r.t. u we get



Definition: the spacetime interval

$$s^2 = c^2(\Delta t)^2 - (\Delta x)^2 \underbrace{-(\Delta y)^2 - (\Delta z)^2}_{\text{IN HIGHER DIMENSIONS}}$$