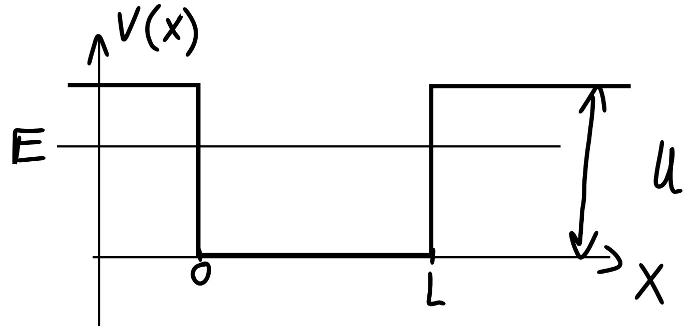
Last time: the finite square well, with potential as drawn:



As long as the particle's energy E < U, the electron is confined to the wire. The bound states can be seen in the bound-state simulation applet. You also studied them in Tutorial 11.

One interesting feature, which was also true for the infinite square well is that the lowest energy is not zero - the electron always has some kinetic energy - this is called the ZERO POINT ENERGY and is a consequence of the uncertainty principle (kinetic energy can only be zero if the momentum is exactly zero, but if we know the momentum, we cannot know anything about the position, which here we do - the electron is mostly inside the wire!) See PS 11, Q1.

-> clicker question

-> clicker question

Another interesting feature in the finite square well is that:
- inside the wire, the wavefunctions have sinusoidal behaviour: they are stading waves, just like for the infinite square well
- since the wavefunction is not forced to be zero at the ends of the wire, it is not zero there
- outside the wire, the wavefunction decays exponentially and is small, but is not zero:
  THERE IS A FINITE CHANCE OF DETECTING AN ELECTRON OUTSIDE THE WIRE (i.e. outside the classically allowed region)

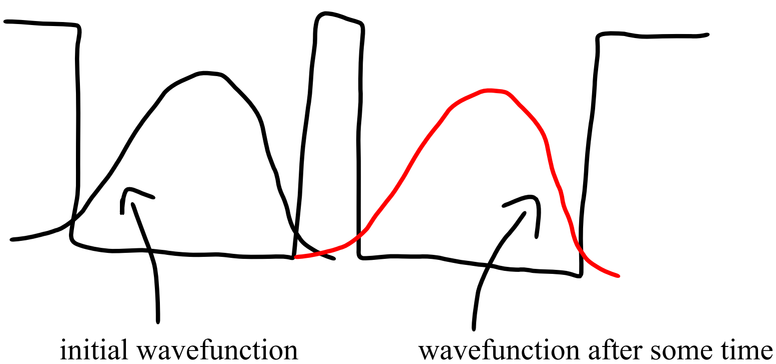
Last time, we saw that in a finite square well, the wavefunction 'stuck outside' the classically allowed region.

Outside the wire, the wavefunction goes like $e^{\pm\sqrt{\frac{2m}{\hbar^2}(U-E)}\,x}$ as we saw in Tutorial 11

We can write this as $e^{\pm x/\eta}$ where η is the penetration distance, given by $\eta = \frac{\hbar}{\sqrt{2m(U-E)}}$

The bigger η, the further outside the well the wavefunction extends (demo this with the bound state simulator)
-> clicker question
The deeper inside the well the particle is, the faster the wavefunction decays outside the allowed region

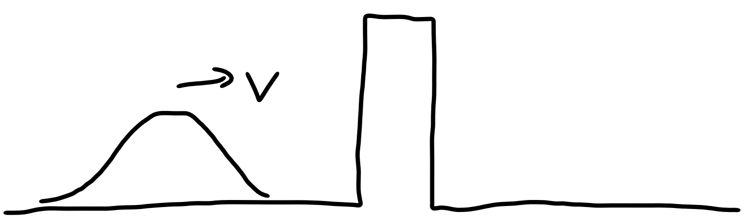
Now, let's consider a situation where there are two wells, with a barrier between them



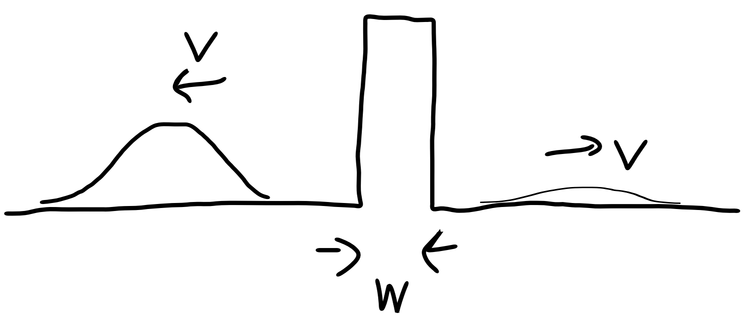
(see this in the bound state simulation)

This is called tunneling: classically, the particle cannot get over the barrier to the other side, but since its wavefunction penetrates through, the particle will eventually get through.

Another demonstration of this is with wavepackets:
A low energy wavepacket approaches a barrier. A small portion of it makes it to the other side:



Before



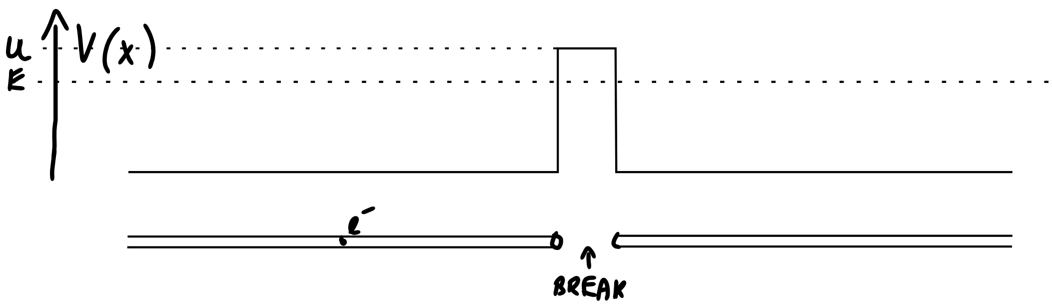
After

You can see this in the wavepacket simulation.

The probability of making it through is proportional to $|\psi|^2$ and since ψ itself decreases like $e^{-w/\eta}$ across a barrier of width w,

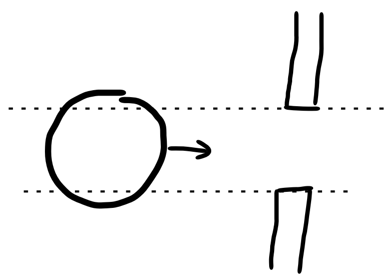
$$P = \left(e^{-w/\eta}\right)^2 = e^{-2w/\eta}$$

-> clicker question (explain examples below first to make the question clear)
Example 1: If a wire has a break in it, the potential energy for an electron looks like this:



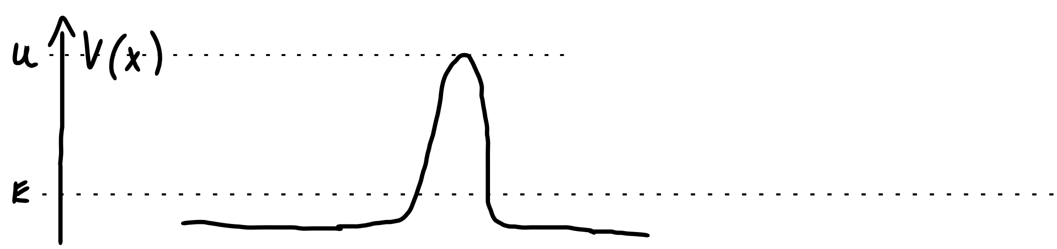
Classically an electron inside the wire with E<U cannot get over the break. In QM, it can!
Current can flow across a broken wire.
This is the principle behind the Scanning Tunneling Microscope (book pg 1292).

Example 2: An atom and a slightly-too-small hole:

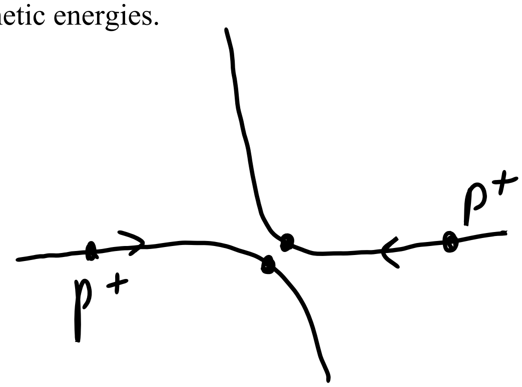


To get through the hole, the atom would have to be deformed (squished) - this costs a lot of energy.

The kinetic energy of the atom is most likely not enough to overcome this potential barrier, but the atom can still tunnel through



Example 3: Two protons moving towards each other. Classically, they will repel and never touch, unless they have HUGE kinetic energies.



But quantum mechanically, they can touch even at low energies.
This is important for the Sun: in the Sun, protons collide to make Deuterium
If they could not tunnel through they electrostatic repulsion, they would never collide. Tunneling is necessary for fusion in the Sun.

$$p^+ + p^+ \to \underbrace{D}_{\text{a bound state of a proton and a neutron}} + e^+ + \nu_e$$