We have been talking about energy eigenstates - these are stationary states (the probability density does not change in time) with a well-defined energy.

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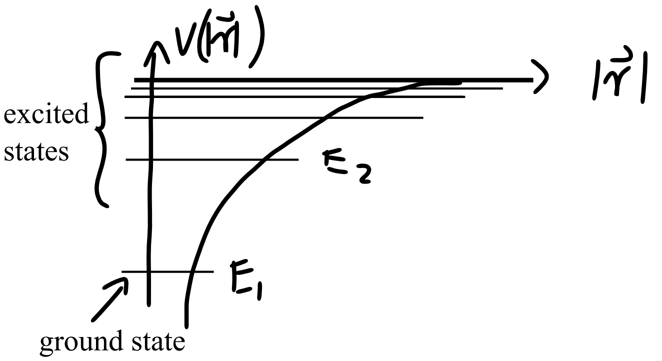
A: it depends! the question did not specify the potential and the energy eigenstates are different for different potentials.

An important example: the hydrogen atom.
Coulomb potential.   3D.

$$-\frac{\hbar^2}{2m}\left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\psi_E - \frac{1}{4\pi\epsilon_0}\frac{e^2}{|\vec{r}|}\psi_E = E\,\psi_E$$

The spectrum is discrete, and has a state with lowest energy (see simulation)



$$E_n = -\frac{13.6\,eV}{n^2}$$

The existance of a lowest energy state implies stability - the atom cannot decay since the eletron has no lowes state energy to go to. This resolves the puzzle of atomic stability. An atom in lowest energy state cannot radiate.

If the atom radiates or absorbes energy, the electron needs to jump from one energy level to another (not allowed in between), so only certain frequencies are permitted.

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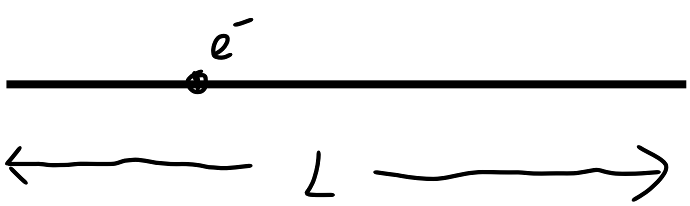
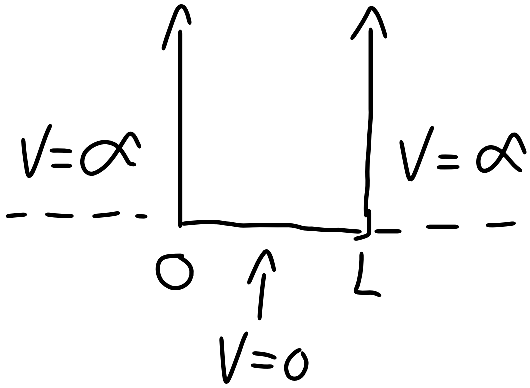
Another example: the inifite square well - electron confined to a finite length wire



$$\begin{cases} \psi_n = \sin\dfrac{n\pi x}{L} \\ E_n = \dfrac{\hbar^2}{2m}\left(\dfrac{\pi n}{L}\right)^2 = \dfrac{h^2 n^2}{8mL^2} \end{cases}$$

You solved this problem in the tutorial. The solutions are
Our bound-state simulation will show us what these look like

The potential for this example is



(Hence the name inifinite square well.)

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Inside the well, away from the walls, the electron is free. So maybe the wavefunction should be a superposition of free matter waves? Actually, it is

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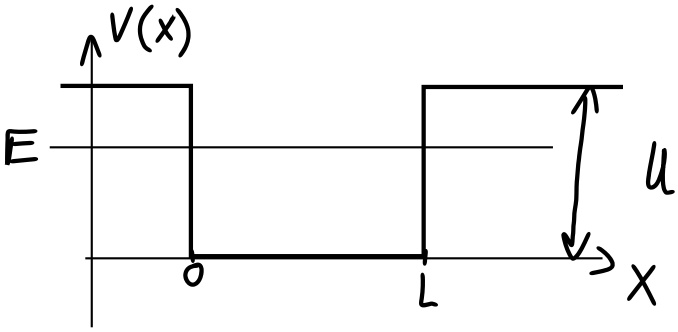
We have a combination of a wave moving to the left and a wave moving to the right

$$\frac{1}{2i} e^{i\frac{n\pi x}{L}} - \frac{1}{2i} e^{-i\frac{n\pi x}{L}} \qquad \text{compare with} \qquad e^{ipx/\hbar}$$

$$p = \frac{n\pi\hbar}{L} = n\frac{h}{2L} \qquad\qquad p = -n\frac{h}{2L}$$

A combination of a left-moving wave and a right moving wave is a standing wave: a stationary state!

In real life, the potential cannot be infinite. A wire is better described with a finite potential:



As long as the particle's energy E < U, the electron is confined to the wire. The bound states can be seen in the bound-state simulation applet. You also studied them in Tutorial 11.

The most interesting features of the stationary states are:

- inside the wire, the wavefunctions have sinusoidal behaviour: they are stading waves, just like for the infinite square well
- since the wavefunction is not forced to be zero at the ends of the wire, it is not zero there
- outside the wire, the wavefunction decays exponentially and is small, but is not zero:
  THERE IS A FINITE CHANCE OF DETECTING AN ELECTRON OUTSIDE THE WIRE
- the lowest energy is not zero - the electron always has some kinetic energy - this is called the ZERO POINT ENERGY and is a consequence of the uncertainty principle (kinetic energy can only be zero of the momentum is exactly zero, but if we know the momentum, we cannot know anything about the position, which here we do - the electron is mostly inside the wire!) See PS 11, Q1.