- simulation: generic quantum state in harmonic oscillator potential

Last time: we introduced special solutions to the Schrodinger equation, called STATIONARY states.
They have the form:

$$\Psi(x,t) = e^{-iEt/\hbar}\,\psi_E(x)$$

This state is called stationary because the probability density of finding a particle at x does not change with time

$$P(x,t) = |\Psi(x,t)|^2 = |e^{-iEt/\hbar}\,\psi_E(x)|^2$$

$$= |\psi_E(x)|^2 \leftarrow \text{time independent!}$$

- simulation - the probabitity density for one eigenstate (2)
-> clicker question about wavefunctions
-> let the simulation answer it. $\mathrm{Re}\,\Psi \sim \cos\frac{Et}{\hbar}$ $\mathrm{Im}\,\Psi \sim \sin\frac{Et}{\hbar}$
General time dependence, as shown before, is a result of adding several of these stationary states togother (simulate again)

To see whether such states actually exist and what ψ_E is, we must substitute the above into the Schrodinger equation.

$$i\hbar\frac{\partial}{\partial t}\left(e^{-iEt/\hbar}\,\psi_E(x)\right) = E\,e^{-iEt/\hbar}\,\psi_E(x)$$

$$\| $$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial}{\partial x^2} + V(x)\right] e^{iEt/\hbar}\,\psi_E(x)$$

Cancel the time-dependent terms and get

$$-\frac{\hbar^2}{2m}\frac{\partial}{\partial x^2}\psi_E(x) + V(x)\,\psi_E(x) = E\,\psi_E(x)$$

This is the time-independent Schrodinger equation. (compare with equation 41.1 on page 1263 of the book)

The stationary state is called an ENERGY EIGENSTATE because it has a well defined energy E associated with it.

By solving the time-independent Schrodinger equation, we solve the whole problem. But as any differential equation, it needs boundary conditions. What are they?

Of special interest are wavefunctions for an electron 'trapped' in the potential, so called bound states
-> clicker question

Physical and boundary conditions for the wavefunction of a bound state:
- continuous (and if possible, the derivative continuous as well)
- goes to zero at infinity
- can be normalized
- is zero in regions where it is physically impossible for the electron to be at all

A general property is that physical solutions to the SE for a bound state are only allowed for some energies and not others. The allowed energies are discrete.
-> see different spectra in the simulation

General recipe for QM problems:

- figure out the potential energy
- figure out the boundary conditions
- solve SE to find energy eigenstates and the allowed energies (the SPECTRUM)
- most general solution is a superposition of energy eigenstates

The spectrum determines a lot of the physical properties of the system.