Last time: the Schrodinger equation for a free particle free = with no forces acting on it

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$$

If you know the wavefunction ψ at some time t, you can compute the time derivative: $\frac{\partial}{\partial t}\psi = \frac{i\hbar}{2m}\frac{\partial^2 \psi}{\partial x^2}$

and therefore compute the wavefunction at some slightly later time t+Δt from $\psi(t+\Delta t) \simeq \psi(t) + \Delta t \frac{\partial \psi}{\partial t}$

The Schrodinger equation is LINEAR: this means that if you have two solutions:

$$i\hbar \frac{\partial \psi_1}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi_1}{\partial x^2} \qquad (1)$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi_2}{\partial x^2} \qquad (2)$$

their superposition is also a solution: $\psi = a\psi_1 + b\psi_2$ solves the equation.

(a and b are any complex numbers)

To prove this, just look at the equation a×(1) + b×(2)\

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The Schrodinger equation tells us about the motion of a quantum particle with no forces acting on it.
It replaces Newton's first law. What about the second law?

Recall where we got this equation from:

$$E\psi = \frac{p^2}{2m}\psi$$

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$$\downarrow \qquad \downarrow$$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$$

If a particle moves in a potential, its energy is the sum of the kinetic energy and the potential energy:

$$E = \frac{p^2}{2m} + V(x)$$

So, let's replace the term corresponding to the kinetic energy above with the sum of kinetic and potential energies:

$$i\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m} + V(x) \right) \frac{\partial^2 \psi}{\partial x^2}$$

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\,\psi}$$

This is the full form of the time-dependent Schrodinger equation.

In three dimensions, we have similarly: $\psi = \psi(x, y, z, t)$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) + V(x,y,z)\,\psi$$

Let's make sure we remember what a potential is:
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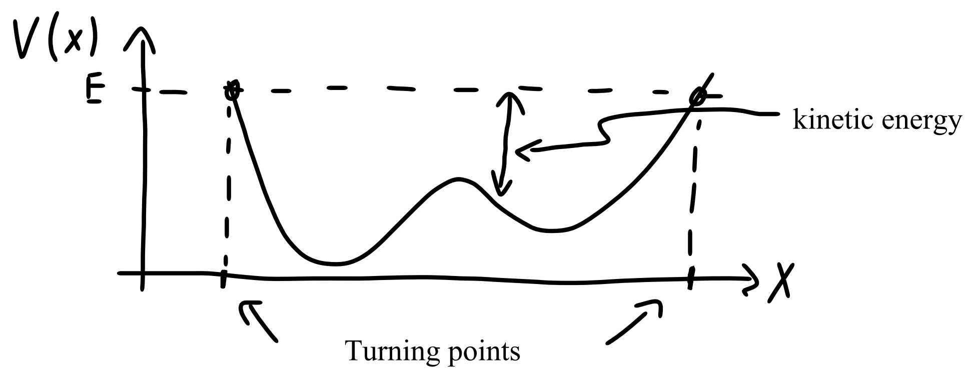
In classical physics, total energy E is conserved and a particle can move in a potential, turning around at the points where its potential energy is equal to the total energy (see figure 41.2 pg 1265 in the book if you need a refresher)



What is the equivalent state in QM? we want a state with a constant, well defined energy. Recall that $i\hbar \frac{\partial}{\partial t}$

$e^{-iEt/\hbar}$

corresponds to the energy (it's called the ENERGY OPERATOR). If the wavefunction as a factor of iin it, the energy operator will just pull down a factor of E, indicating that we have a well defined energy state:

$$i\hbar \frac{\partial}{\partial t}\left(e^{-iEt/\hbar} \psi_E(x) \right) = E \, e^{-iEt/\hbar} \psi_E(x)$$

arbitrary x-dependence of the wavefunction

A wavefunction of the form $e^{-iEt/\hbar} \psi_E(x)$ is called a stationary state. Why?

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A stationary state's time dependence is just a complex over-all phase: it barely changes anything. The probability density is the same for all times, for example.

Next up: the time-independent Schrodinger equation.