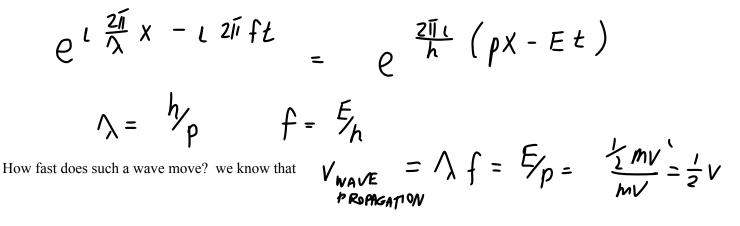
In the tutorial yesterday (Tutorial 10), you saw that wavepackets evolve: they move, they spread, they change shape.

A simple de Broglie wave has simple time dependence. It is given by the de Broglie frequency:



The wavefronts move with half the expected speed. This is known as PHASE VELOCITY.

As you saw in the tutorial, the above formula for f ensures that **wavepackets** with average momentum p move with speed v=p/m. The wavepacket moves with the same speed the particle does. The speed of the wavepacket is called the GROUP VELOCITY.

-> look at the simulation of wave packets to see that the phase velocity is smaller than the group velocity.

Since we know that all wavefunctions can be expanded as a sum over planewaves, this is enough to figure out how any wavefunction changes with time:

Say, at t=0, we have

$$\Psi(x, t=0) = \int \tilde{\Psi}(p) e^{i\frac{2\pi}{h}xp} dp$$

QM is a **linear theory**: the principle of superposition holds. This means that each component of the wave evolves in time independently of all the other componets. Therefore:

$$\Psi(x,t) = \int \widetilde{\Psi}(p) e^{\frac{2\pi i}{h}(xp-Et)} dp$$

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How does the momentum wavefunction change?

$$\Psi(x, t) = \int \widetilde{\Psi}(p, t) e^{-\frac{1}{h}xp} dp$$

=> $\widetilde{\Psi}(p, t) = \widetilde{\Psi}(p) e^{-\frac{2\pi}{h}Et}$

A useful way to describe the time evolution of a function is often through a differential equation. Start with the following observation:

$$E = \frac{1}{2}MV^2 = \frac{1}{2}\frac{p^2}{m}$$

Notice that you can 'pull down' a factor of E from a planewave by acting on it with a time derivative

$$\begin{aligned} \frac{\partial}{\partial t} e^{-i \frac{\lambda \pi}{h}} \frac{Et}{Et} &= -i \frac{2\pi}{h} Ee^{-i \frac{\lambda \pi}{h}} Et\\ i \pm \frac{\partial}{\partial t} e^{-i \frac{Et}{\hbar}} &= E e^{-i \frac{Et}{\hbar}} \\ \frac{\partial}{\partial t} e^{-i \frac{Et}{\hbar}} &= E e^{-i \frac{Et}{\hbar}} \end{aligned}$$
You can also pull down a factor of p with an x-derivative:
 $-\iota \pm \frac{\partial}{\partial X} e^{i \frac{\lambda \pi}{\hbar}} &= \rho e^{i \frac{\lambda \mu}{\hbar}} \end{aligned}$
So, we can write
 $E e^{i \frac{\pi}{\hbar} (xp - Et)} &= \frac{i}{2\pi} p^2 e^{i \frac{\pi}{\hbar} (xp - Et)}$
 $i \pm \frac{\partial}{\partial t} e^{i \frac{\pi}{\hbar} (xp - Et)} &= -\frac{\pi}{2\pi} \frac{\partial}{\partial x^2} e^{i \frac{\pi}{\hbar} (xp - Et)}$

By linearity, then:

$$i\hbar\frac{\partial}{\partial t} \Psi(x,t) = -\frac{\hbar}{2m} \frac{\partial}{\partial x^2} \Psi(x,t)$$

This is the (time-dependent) Schrodinger equation for a free particle. What does this equation tell us? we can write it as

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$$\frac{\partial}{\partial t} \psi = \frac{i\hbar}{2m} \frac{\partial^2_x \psi}{\partial^2_x \psi}$$

$$\psi(x, t+\Delta t) \simeq \psi(x, t) + \Delta t \frac{\partial^2_t \psi}{\partial t} (x, t) = \psi(x, t) + \frac{i\hbar \Delta t}{2m} \frac{\partial^2_x \psi}{\partial^2_x \psi}$$

You can determine the wavefunction at position x and time t+ t from the properties of the wavefunction at time t and -> clicker question position x

We already know what happens to wavepackets under this equation: they move with a group velocity equal to the average momentum divided by mass, they spread and even change shape.

-> clicker question

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