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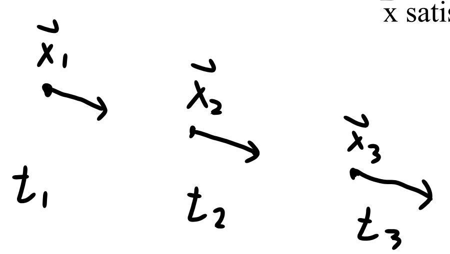
But, what is the wavefunction?
A: It's a complete description of the state of the particle.

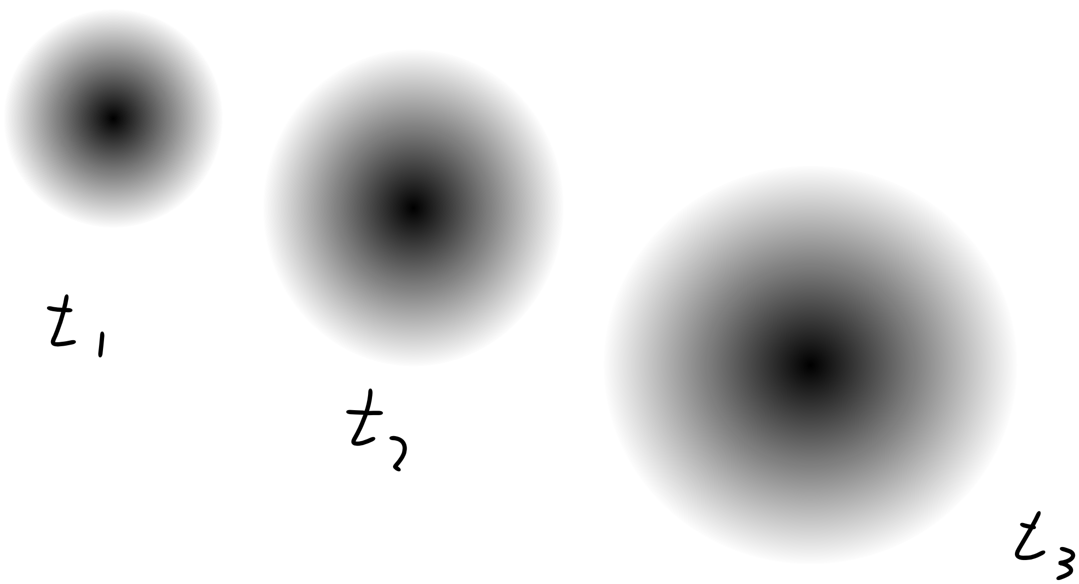
To describe a particle in classical mechanics, we specify positions and velocities. These can change with time.

$\vec{x}$ satisfies Newton's equation:

$$m \frac{d^2\vec{x}}{dt^2} = \vec{F}$$



To describe a particle in QM, we specify the wavefunction. The wavefunction changes with time, too!
(Schrodinger equation tells you how, we will get to this.)



In QM, the wavefunction replaces positions and velocities in the description of a particle's evolution. Given a wavefunction, we can predict the probabilities of detecting the particle at different positions, or of measuring it to have a particular velocity.

Looking ahead: Schrodinger equation:

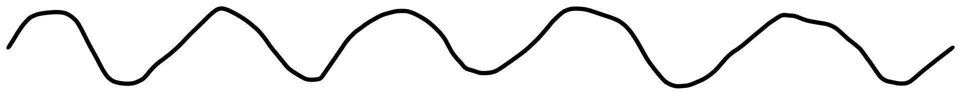
$$i \frac{d}{dt} \psi(x,t) = -\frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t) + V(x)\,\psi(x,t)$$

Let's start by considering the motion of a free particle, an electron with momentum p. Diffraction experiments confirm de Broglie's guess that the wavelength of such an electron is $\lambda = \frac{h}{p}$

At a given time, the wavefunction should be a wave with this wavelength. Assume complex:

$$\psi(x) = Z\, e^{i\, 2\pi x/\lambda} = Z\, e^{i \frac{2\pi}{h} p x}$$

To picture it, just draw the REAL part: $|Z| \cos\left(\frac{2\pi}{h} p x\right)$



But, notice that the real part is the same for p and -p, so it cannot contain all the information about the electron (in particular, it does not tell you which way the electron is going)

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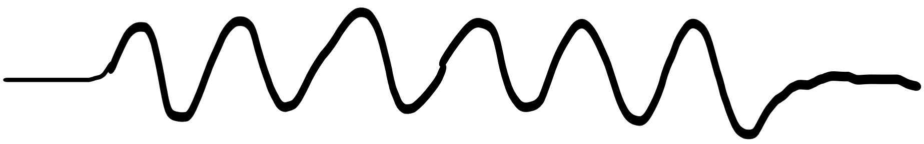
So, the probability density is basically a constant: $|Z|^2$ This means the total probability is infinite:

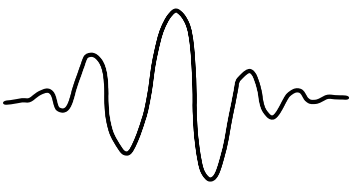
$$\int_{-\infty}^{\infty} P(x)\, dx = \int_{-\infty}^{\infty} |\psi(x)|^2\, dx = \int_{-\infty}^{\infty} |Z|^2\, dx =$$ inifinity!

But, the wavefunction we wrote down is a bit unrealistic - it says that the particle is equally likely to be found anywhere in the whole world. In practice, the particle's wavefunction has a finite extent



and you can normalize it.
We will use the unnormalized inifinite wave for convenience a lot, though. There are ways to make the math come out OK.

An electron sometimes behaves like a particle, with a pretty well defined position. This corresponds to a wavepacket with a smaller extent:



It turns out that this wavepacket can be writen as a quantum superposition of a whole bunch of plain waves.
(DEMO) http://phet.colorado.edu/en/simulation/fourier
- show that you can make different shapes by combining different frequencies
- show the 'beat wavepacket' by combining a 4th and 5th harmonic
- show the pre-programmed wavepacket, and that there is a whole train of them
- show how you can make just one if you go to the continuum
- finally, show how the width of the wavepacket and the spacing of the fringes varies with the position and width of the wavepacket in momentum-space

We can define $\psi(p)$ to be the amount of wave with wavelength $\frac{h}{p}$ in the superposition

$|\psi(p)|^2$ is then the probability densitity for finding a value p if we measure momentum

We have a new measurement (momentum), new eigenstates (the plane waves, which have definite momentum) and we can write the state of the particle as a quantum superpositiono of these eigenstates, with coefficients giving us probabilities of different measurement outcome. This is parallel to the situation with position.