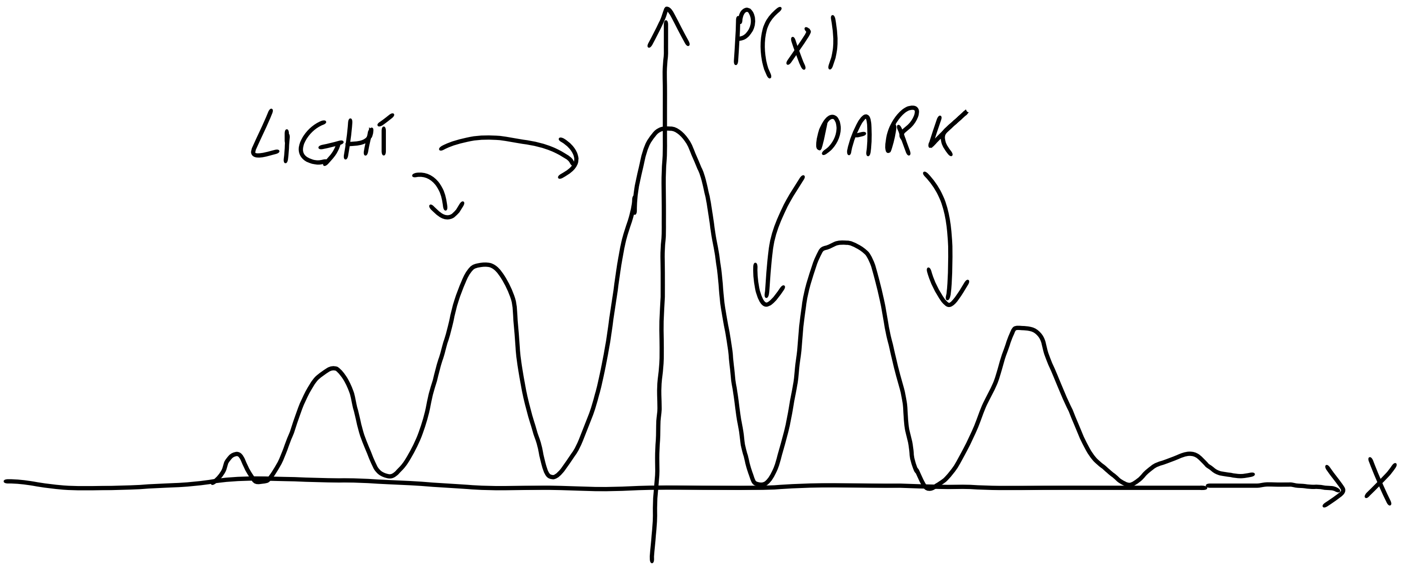
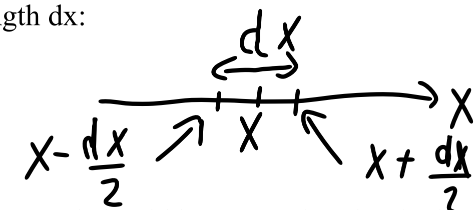
Last time we established that to explain the existance of interference fringes in a double-slot experiment where there is only one photon in the experiment at a time, we needed to assume that the photon is in a delocalized quantum state:

A delocalized quantum state interacts with both slots (the photon 'goes through' both slots), which alters it. The altered quantum state approaching the screen encodes in it the interference pattern is such a way that the probability of a photon hitting the screen at any given point is proportional to the classical intensity:

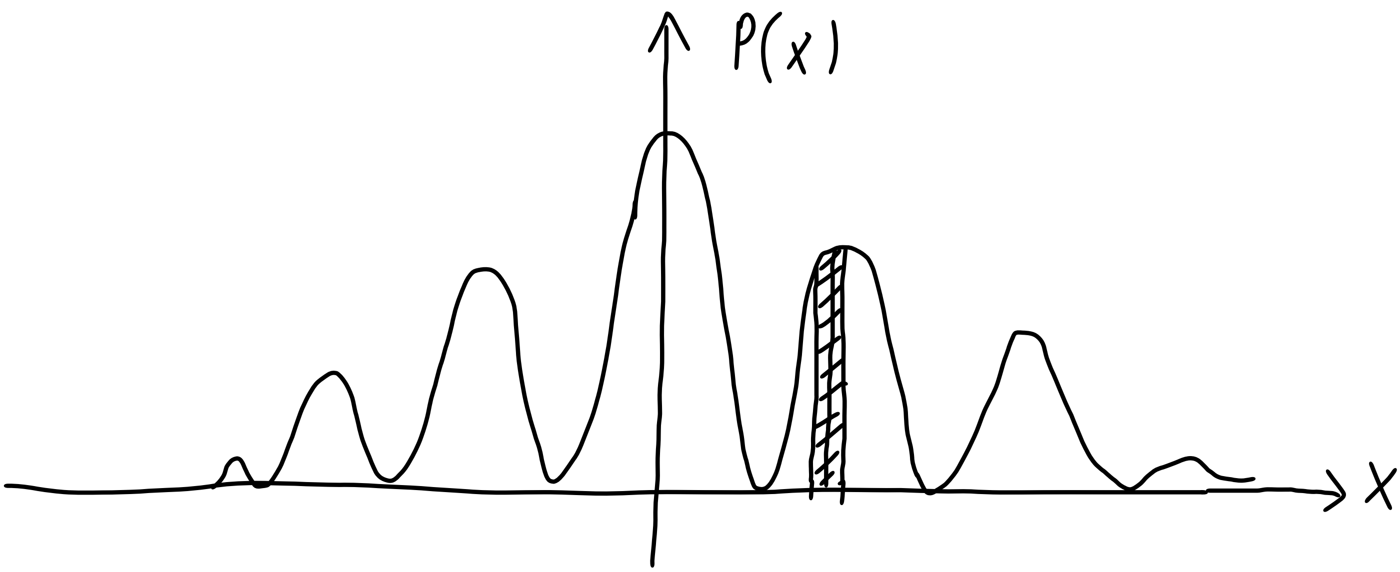


Actually, we need to be a bit more careful. The probability of a photon hitting a particular exact point at some x coordinate is zero, since poinst are inifinitely small. Instead, we can have the probability of finding the photon near some x, say between x-dx/2 and x+dx/2, in an interval of length dx:



The probability of being found in such a small interval should be proportional to its length (dx) and to the local 'probability density': P(x)

$$\text{PROP}\left(\left(x-\frac{dx}{2},\ x+\frac{dx}{2}\right]\right) = P(x)\,dx$$

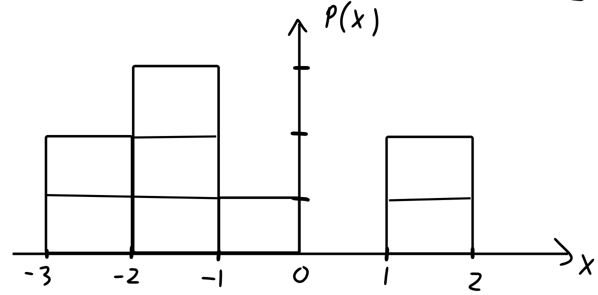


More generaly, the probability of being found anywhere in the interval from x_1 to x_2 is just the area under the curve, or the integral of the probability density P(x) from x1 to x_2

$$P(\text{FOUND IN } [x_1, x_2]) = \int_{x_1}^{x_2} P(x)\,dx$$

The probability of being found anywhere at all must be 1, so: $\int_{-\infty}^{\infty} P(x)\, dx = 1$ (normalization)

-> clicker question



The total area is 8 units and area under the block from x=1 to x=2 is 2 units, so probability = 2/8 = 1/4

Note: the relationship between probability and probability density is analogous to that between mass and density. In 3D, the probability of being found within some volume V is

$$\text{PROB}(\text{IN } V) = \int_V P(\vec{x})\, d^3x$$

The suprising thing is that this sort of interference pattern is observed not only with photons, but also with matter particles: electrons, atoms and even molecules! (see slide for interference fringes with the buckyball molecule).

Apparently, ordinary particles must have some sort of wave associated with them to produce the interference pattern: these are called MATTER WAVES.

De Broglie postulated that matter waves have wavelength determined by the momentum of the particle: $\lambda = h/p$
which agrees with de diffraction experiments on electrons and larger particles.
-> clicker question

What is the connection between matter waves and quantum states?

Let's look at a delocalized quantum state again. It is a quantum superposition of the eigenstates at different positions:

$$z_{x_1} |x_1\rangle + z_{x_2} |x_2\rangle + z_{x_3} |x_3\rangle + \dots$$

What if I wanted to write down a quantum state which includes everypoint in space? I would need to sum over all possible positions. This is what an integral is for! so, we get a quantum state like this

$$\int d\vec{x}\ \psi(\vec{x})\ |\vec{x}\rangle$$

where $\psi(x)$ is the function describing all the different coefficients for positions at different points (the analog of the z_i's in the previous expression).

$\psi(x)$ is called the **wavefunction**. It is a convenient way to write a quantum state for a delocalized particle.

Before, the probability was given by coefficients squared. Now, the probability is given by the square of the wavefunction:

$$P(x) = |\psi(x)|^2$$

Two things about the above formula:

- the absolute value signs mean MAGNITUDE (which we will need if the wavefunction is complex)
- because the probability must be normalized to 1, we need to normalize our wavefunction as well

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1 \quad \underline{\text{OR}} \quad \int |\psi(\vec{x})| \, d\vec{x}^3 = 1 \quad (3D)$$

Looking ahead: the wavefunction for a particle with momentum p is (according to de Broglie)

$$\lambda = \frac{h}{p} \qquad e^{i(2\pi\, x/\lambda)} = e^{i\left(\frac{2\pi}{h} x p\right)}$$

(This is just the dependence on x. We also need dependence on t - we will cover this later)
This is just like the electromagnetic wave, so it's not surprising that there are fringes.
(slide)

The fact that all particles (photons, electrons, phonons, atoms, whatever) exibit both particle-like behaviour (leave a point-like mark when hitting the screen for example) and wave-like behaviour (diffraction fringes) is called wave-particle duality.

We will look at it as follows: particles have wavefunctions associated with them. Delocalized wavefunctions can lead to wave-like behaviour (eg, diffraction). When we try to measure the position of a delocalized particle, however, it randomly choses a localized quantum state to be in and we get a well defined position measurement. The probability of being measured at a given point is determined by the square of the wavefunction at that point.

-> clicker question