We have talked about the experimental evidence that light is actually made up of discrete particles: photons. But there is also plenty of evidence that light really is a wave. Diffraction is probably the most convincing.

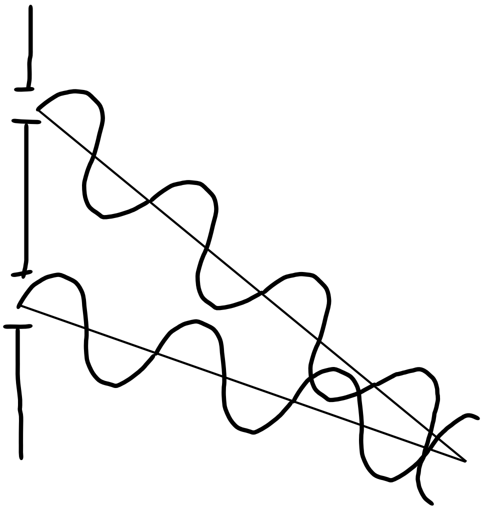
Diffraction occurs when two or more wavefronts come to the same point via paths of different length.

If (path 1 - path 2) = $N\lambda$, you get constructive interference and a bright fringe

$N \in \mathbb{Z}$

If (path 1 - path 2) = $(N+\frac{1}{2})\lambda$, you get destructive interference and a dark fringe

Example: dark fringe due to path 1- path 2 = $\lambda/2$



What if we only put in one photon at a time? (see pictures in the clicker questions).
Every photon individually knows about the interference!
Wave-particle duality is the term often used to describe the strange fact that photons behave as both particles (hit the screen at a well defined certain spot) and waves (interference fringes do occur).

-> clicker question

Quantum explanation

The photon approches the two slits in a quantum state. Going through the slits modifies this state. When it gets to the screen, the quantum state 'knows' about the wave interference and the resulting intensity. When a photon falls on the screen and its position is detected, we have a position measurement.

What are the eigenstates? Let's assume that there is a quantum state describing photon at position x along the screen:

$$|x\rangle$$

Now, let's assume that the quantum state of the photon as it approaches the screen is a quantum superposition of many states at different positions:

$$z_1|x_1\rangle + z_2|x_2\rangle + z_3|x_3\rangle + \ldots$$

The photon will be detected at x_1 with probability $|z_1|^2$ etc....

How are the coefficients and therefore probabilities determined? they must agree with the wave picture, so the probability must be proportional to intensity of light at this point

$$\mathrm{PROB}(x_1) \propto I(x_1) \quad \mathrm{OR} \quad |z_1|^2 \propto I(x_1)$$

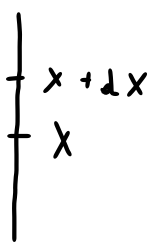
-> clicker question

Does the photon go through just one slot, or both?
New experiment, cover up one slot at a time and let an individual photon through. Everytime chose a random slot (based on a coin toss).
-> clicker question
The pattern dissapears.
To produce the fringes, each individual photon must go through both slits!
It approaches the slits in a quantum superposition (not localized), with componets near both slits:

$$a_1 \,|\text{SLIT } 1\rangle + a_2 \,|\text{SLIT } 2\rangle$$

So, each photon's quantum state goes through both slits, then interfers with itself.

Talking about probabilities of being at a certain point is not quite precise enough.
The probability of being detected at a point is zero, since a point is infinitesimaly small.
We should be talking about a probability of being detected on a small interval:



probability of being in the small interval dx = $P(x)\,dx$

Probability density

The probability density is a function P(x) such that the probability of being detected between x_1 and x_2 is

$$P([x_1, x_2]) = \int_{x_1}^{x_2} P(x)\,dx$$

To agree with experiment, we must have that $P(x) \propto I(x)$ for photons.