IN TUTORIAL WE SAW THAT

$$p_x = m\gamma v_x = m\frac{\Delta x}{\Delta t/\gamma} = m\frac{\Delta x}{\Delta \tau}$$ PROPER TIME

$$E = mc^2\gamma = mc^2\frac{\Delta t}{\Delta t/\gamma} = mc^2\frac{\Delta t}{\Delta \tau}$$

OR $$E/c^2 = m\frac{\Delta t}{\Delta \tau}$$

m, τ ← INVARIANT, SO

$$\Delta\tilde{x} = \gamma(\Delta x - v\Delta t) \Rightarrow \boxed{\tilde{p}_x = \gamma\left(p_x - v\frac{E}{c^2}\right)}$$

$$\Delta\tilde{t} = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) \Rightarrow \frac{\tilde{E}}{c^2} = \gamma\left(\frac{E}{c^2} - \frac{v}{c^2}p_x\right)$$

$$\boxed{\tilde{E} = \gamma(E - v p_x)}$$

-> clicker question

$$E^2 - (cp)^2 = \text{INVARIANT}$$

$$(mc^2)^2\frac{1}{1-u^2/c^2} - m^2c^2\frac{u^2/c^2}{1-u^2/c^2}c^2 = (mc^2)^2 \checkmark$$

Because the rest mass is the same in all reference frames (it's just the mass), this combination is invariant, the same in all reference frames.

CAUTION: DIFFERENT γs! BETTER USE $\gamma(u)$
$\gamma(v)$

$$E^2 - c^2p^2 = (mc^2)^2 = m^2c^4 \quad \text{vs} \quad c^2(\Delta t)^2 - (\Delta\vec{x})^2 = s^2$$

"length" squared of (E, p_x, p_y, p_z)

"length" squared of (Δt, Δx, Δy, Δz)

If you know p, you can compute E and vice versa, as long as you know m.

If you know both p and E (from measurement), you can reconstruct m!

-> Clicker question

BULLET IMPARTS $(6.67\text{g})c$ MOMENTUM (mv)
AND $(8.33\text{g})c^2$ ENERGY

$$m = \sqrt{E^2/c^4 - p^2/c^2} = \sqrt{8.3^2 - 6.67^2} = 5\text{g}$$

What about the kinetic energy of the car?

$$\frac{1}{2}(2000\text{kg})(1000\,\text{m/s})^2 = 2 \cdot 10^9\ \text{J}$$

$$(8.33\text{g})c^2 = 7.5 \cdot 10^{14}\ \text{J}$$

↑

NEGLIGABLE