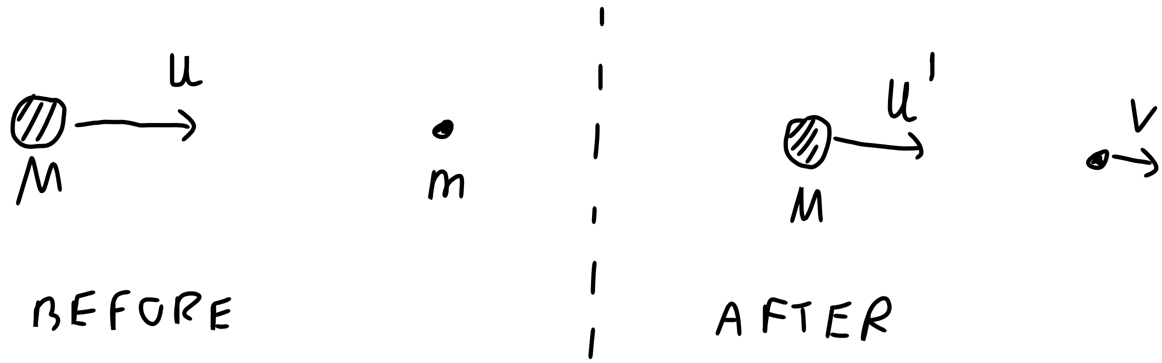
Collisions, momentum and conservation of momentum



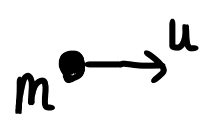
Conservation of momentum: $$Mu = Mu' + mv$$

Conservation of energy $$\frac{1}{2} M u^2 = \frac{1}{2} M(u')^2 + \frac{1}{2} m v^2$$

rumblings of the universal computation machine.... $$v = \frac{2M}{m+M} u$$

-> clicker question

v can come out greater than c !!!
something is wrong: either conservation of momentum and energy is broken, or we have
the wrong formulas for p and E. Conservation of momentum is such a fundamental law we will no abandon it.



$$\begin{cases} E = \gamma_p mc^2 \\ p = \gamma_p mu \end{cases} \quad \text{WHERE} \quad \gamma_p = \frac{1}{\sqrt{1 - u^2/c^2}}$$

m is the rest mass - the mass of the particle as measured in its own frame of reference
(i.e. at rest)

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$$p = \frac{mu}{\sqrt{1-u^2/c^2}} \qquad \text{SINCE } \sqrt{1-u^2/c^2} < 1 \quad , \quad p > mu$$

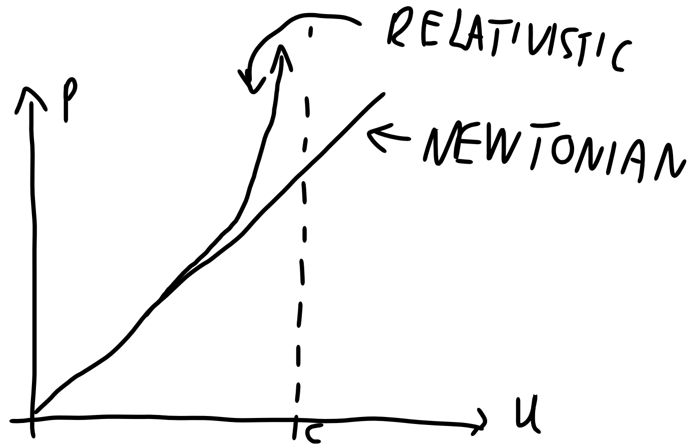

FIG 37.34
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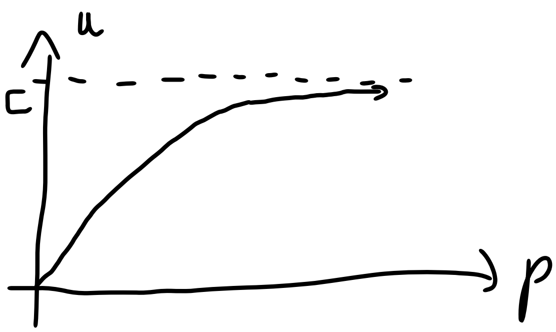
A particle's momentum can increase arbitrarily, but the velocity is bounded by c

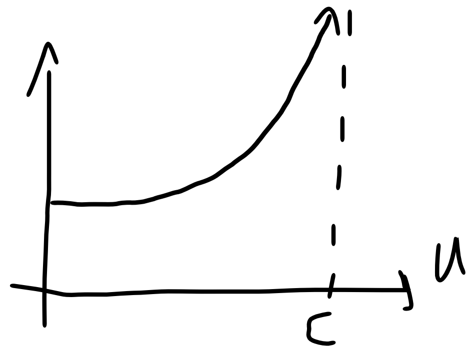


Newton's second law still holds in the form : $F = \dfrac{dp}{dt}$

(not the same as F=ma!)

A constant force can give a particle a momentum which increases for ever - but the velocity just approaches c.
No object with a nonzero rest mass m can ever move at c, since you would have to accelerate for ever.

Energy. It can also increase without bound:



Let's look at it more closely for small u:

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} \simeq mc^2\left(1 + \frac{1}{2}\frac{u^2}{c^2} + O(u^4)\right)$$

$$= \underbrace{mc^2}_{\text{Rest mass energy}} + \underbrace{\frac{1}{2}mu^2}_{\text{Kinetic energy}} + O(u^4)$$

$$E = E_0 + K$$

But the split is not as clear as you would think: it depends on what you consider the internal and the external DofF
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A: rest mass energy is associated with internal degrees of freedom (ones you don't see moving or wiggling)
Other stuff goes into kinetic energy

-> clicker question