Physics 200 Problem Set 9

Due at the end of class, FRI Nov 19th

Practice problems: do not hand in. Textbook pg 1235, exercise 15, pg 1237, problem 45, pg 1259, problems 35, 39. In addition, you can try doing an old tutorial on wavepackets, which uses the same simulation as question 4 of this PS and is posted on the course website under Lecture 27.

1. Light passes through one slot on a way to a screen. The intensity profile on the screen is measured to be

$$\frac{I_0}{\cosh(x)}$$

where I_0 is the intensity at the center of the pattern.

(a) What is the probability that a given photon will hit the screen between $x = 0$ and $x = 1$?

(b) What is the probability that the first four photons will hit the screen at $x > 0$?

Remark 1: The hyperbolic cosine, $\cosh(x)$, is defined as $(e^x + e^{-x})/2$.
Remark 2: The above function does not integrate to 1.
Remark 3: If you are having trouble with the integration, the following substitution might help: $e^x = \tan(y)$.

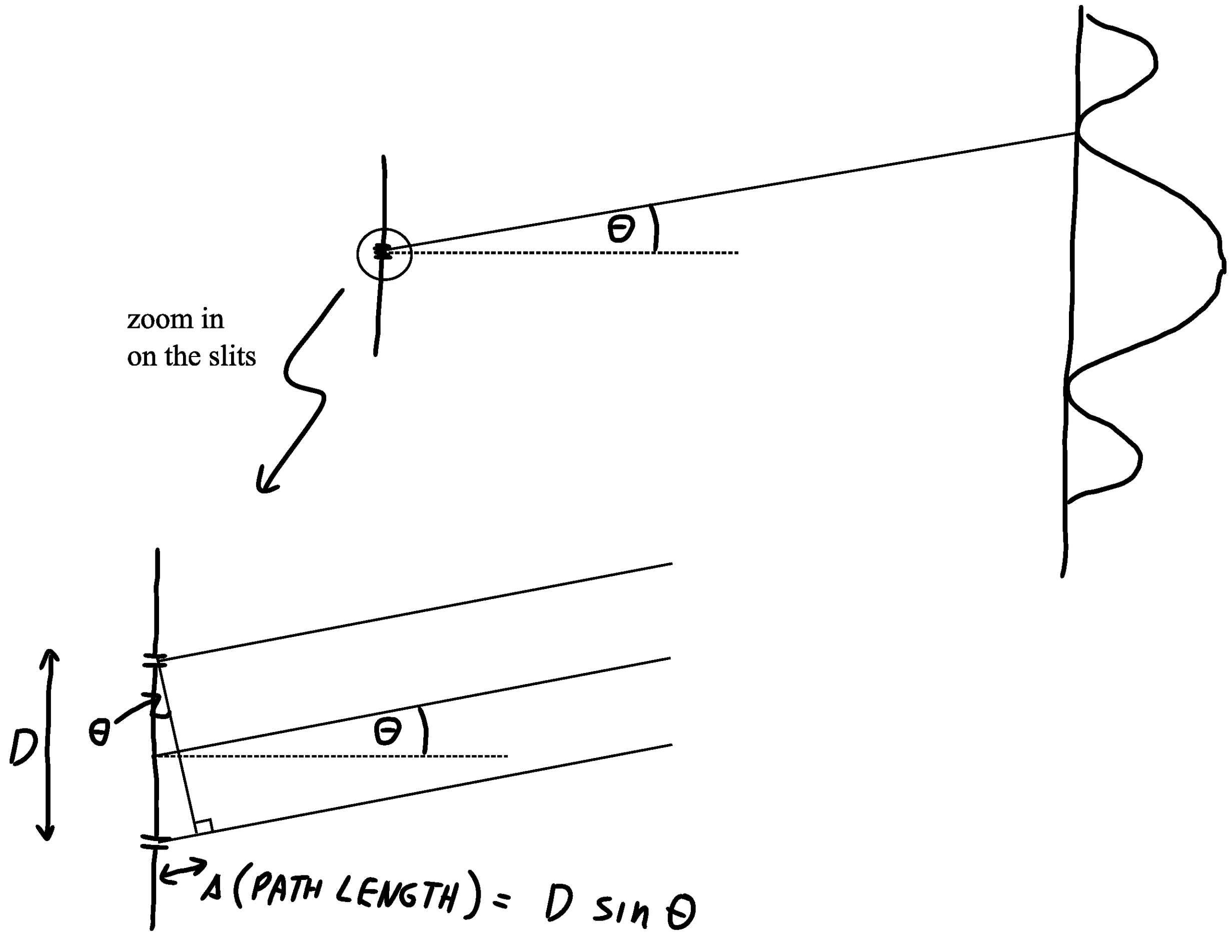
2. You might be familiar with the fact that the resolution of a microscope is ultimately limited by the wavelength of the light used. In this question, we will examine this.

We know that when light passes through small features (such as narrow slots), instead of getting the image of the features on the screen, we get interference fringes. Let's examine how close together two really narrow slits can be before we can no longer resolve them (see that there are two slits and not just one).

(a) Explain, using the figure on the next page as your guide, why the double slit interference pattern gets wider when we make the slits closer together.

(b) Sketch what the pattern looks like on the screen if the first dark fringe is at $\theta = \pi/2$. Can you infer the existence of two slits (as opposed to one slit twice as wide) from this pattern?

(c) For light with wavelength λ, what is the distance D between the slits when $\theta = \pi/2$?



The first dark fringe appears where the difference in path lengths is half the wavelength:

$$D \sin\theta = \lambda/2$$

3. In the previous question, you discovered that you cannot resolve features smaller than the wavelength of the light used to 'view' stuff.

(a) The hydrogen atom has a radius of about 0.5Å. What wavelength of light would you have to use to resolve features in the electron density with size equal to the radius?

(b) What is the energy of one photon of such light (in eV)?

(c) The ionization energy of hydrogen (the amount of energy needed to remove the electron to infinity) is 13.6eV. What do you think would happen if a photon with the energy you computed in part (b) scattered of the electron in hydrogen? Explain the significance of this computation in the context of the Heisenberg uncertainty principle.

(d) Electron waves can also be used to 'see' stuff. Recall that the interference pattern for electrons is determined by the de Broglie wavelength $\lambda = h/p$. Using very fast electrons produced at particle accelerators, physicists have been able to 'see' inside the proton and determine that protons are made out of quarks. To do this, you need resolution of around 10^{-16}m. What energy (in GeV) did the electrons need to have to do this?

4. For this question you will need to use the applet at
http://phet.colorado.edu/en/simulation/fourier
Go to tab 'Discrete to Continuous' at the top and then set Spacing between Fourier components to 0. Also, turn on the 'x-space envelope' and the 'width indicators' so you can see better what is going on. The top plot is drawn as a function of wavevector k, which is related to the wavelength by $k = 2\pi/\lambda$. Explore the behaviour of the plots when moving the other controls around to answer the following questions:

(a) What is the relationship between the width of the wavepacket in momentum space, σ_k, and the width of the wavepacket in position space, σ_x?

(b) Using the above definition of k and the de Broglie wavelength formula, rewrite the relationship you obtained in part (a) as a statement about the uncertainties in momentum and position.

(c) Now set k_0 to 9π and σ_k to roughly 2. You cannot make the slider for k_0 be any smaller, so you will need to extrapolate. Sketch the wavepackets (including the envelope) with

(i) $\sigma_k = 2$ and $k_0 = 3\pi$

(ii) $\sigma_k = 2$ and $k_0 = 0$

If the wavepackets you drew represent a wavefunction of an electron, what is the velocity of this electron in case (ii)?