Physics 200 Problem Set 8

Due at the end of class, Wed Nov 10th

Practice problems: do not hand in. Since material covered this week is not in the book, notes on complex numbers, polarizations and photon polarizations have been posted on the course website (under lectures 22 and 23). As practice, you should do the questions in the Notes on complex numbers, as well as review Tutorials 8 and 9 all the way to the end.

1. Complex numbers are useful for many things. In this question, you will derive some trigonometric identities using the power of complex numbers.

(a) Consider the following fact:

$$e^{i(x+y)} = (e^{ix})(e^{iy})$$

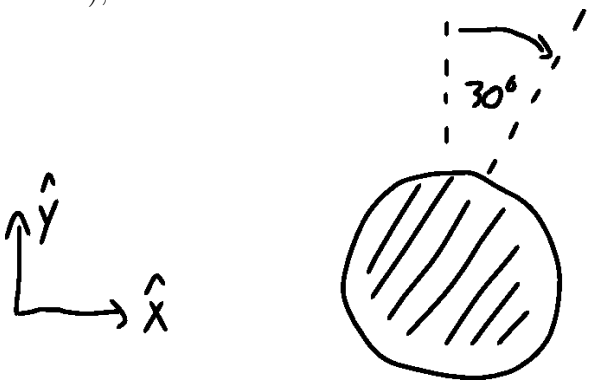
with x and y real. Write $e^{i(x+y)}$, e^{ix} and e^{ix} in the form $a + ib$, then multiply out the RHS. By considering the real and imaginary parts of the resulting equation, derive formulas for $\cos(x + y)$ and $\sin(x + y)$.

(b) Now, consider this equation:

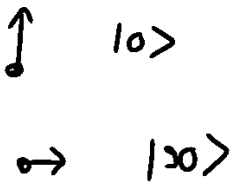
$$e^{3ix} = \left(e^{ix}\right)^3$$

Prove that $\cos(3x) = \cos^3(x) - 3\cos(x)\sin^2(x)$.

2. Consider a polarizer with its axis pointing at an angle of 30° to the y direction (vertical direction), as shown.



Photon quantum states:



(a) Which photon quantum state will be transmitted by this polarizer with 100% certainity? which photon quantum state will be absorbed with 100% certainity? give your answers as quantum superpositions of $|0\rangle$ and $|90\rangle$ and don't forget to normalize properly.

(b) Let's denote the transmitted photon and absorbed photons from part (a) by $|t\rangle$ and $|a\rangle$, respectively. Solve for $|0\rangle$ and $|90\rangle$ in terms of $|t\rangle$ and $|a\rangle$.

(c) Now consider a photon polarized at 135°. What is the probability of this photon going through the polarizer? [Hint: re-write the quantum state for the photon in terms of $|t\rangle$ and $|a\rangle$ using your answer to part (b).]

3. An elliptically polarized photon might have a quantum superposition with complex coefficients.

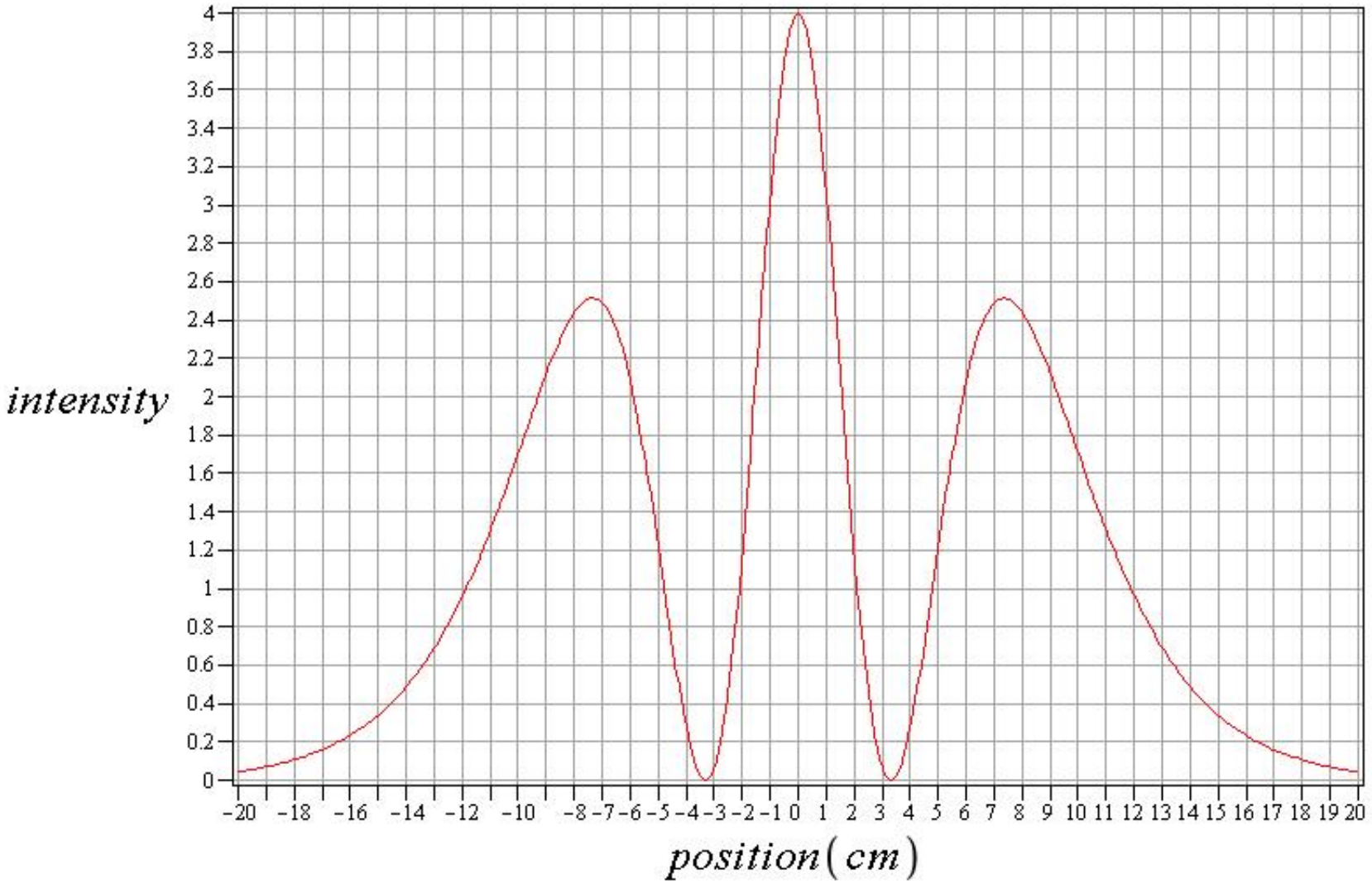
(a) Consider photons described by $(|0\rangle + i|90\rangle)/\sqrt{2}$ and $(|0\rangle - i|90\rangle)/\sqrt{2}$. Explain why these are called right and left circularly polarized photons [Hint: review tutorial 8 and in particular question 1(b).]

(b) What is the probability that a photon in a quantum state $(|0\rangle + i|90\rangle)/\sqrt{2}$ will go through a vertically oriented polarizer?

(c) What is the probability that this photon will go through a polarizer at an arbitrary angle θ? [Hint: this is similar to Question 2 above.]

4. The attached figure shows a diffraction pattern (intensity as a function of distance on the screen) for a two-slit experiment. Using the figure, estimate as accurately as you can the probability that the first two photons both hit within the central interference fringe. (Notice that the area under the curve is not 1.)

Challenge question (not for credit). In the case of the circularly polarized photons from Question 3, describe how you would measure the polarization of a beam of such photons if you had a polarizer and a quarter-wave plate.


intensity
4
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-20 -18 -16 -14 -12 -10 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
position(cm)