Physics 200 Problem Set 5

Due at the end of class, Wed Oct 20th

Practice problems: do not hand in. End of Chapter 37, Problems 41, 69, 73.

1. Consider Newton's second law in relativistic setting:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\,\frac{m\vec{u}}{\sqrt{1-|\vec{u}|^2/c^2}}$$

Consider a particle moving in the x-direction on which a force acts in the y-direction. This causes the y-component of $\vec{u}$ to change, while the x-component stays constant, $u_x = U$. At t=0, $u_y = 0$.

(a) Compute the y-component of the acceleration at the instant $t = 0$ in terms of F, m and $\gamma(U)$.

(b) If you wanted to write a formula of the form $F = Ma$ based on your answer to part (a), what would M have to be?

2. In this question you will verify that length contraction together with time dilation resolves the discrepancy you saw in Question 4 on Tutorial 1. You may use the following facts about electromagnetic forces:

- An infinitely long line of charges with charge density ρ exerts a force $\frac{\kappa\rho Q}{d}$ on a charge Q a distance d away.
- An infinite straight wire carrying a current I exerts a force $\frac{\kappa I v Q}{c^2 d}$ on a charge Q moving with speed v parallel to the wire a distance d away.

κ here is just some constant.

(a) Let a positive charge Q with mass m be positioned a distance d away from an infinitely long line of densely spaced positive charges. We will take the charge density (change per unit length) of the line of charges be ρ. If the charge is initially motionless, what is its initial acceleration?

(b) Consider a frame of reference moving with velocity v along the direction of the wire. Using Lorentz transformations, and your answer to part (a), what is the initial acceleration $\tilde{a}$ in the moving frame of reference?

(c) What is the electric force and the magnetic force in the moving frame? What is the net force $\tilde{F}$?

(d) What is the effective mass $\tilde{m}$ of the charge in the moving frame? (Hint: Use the answer to Question 1 of this Problem Set).

(e) Is Newton's Second law satisfied in the moving frame (ie, do you get that $\tilde{F} = \tilde{m}\tilde{a}$)?

3. A spaceship is equipped with a Total Mass Conversion Drive: a device which can convert the entire mass energy of any fuel into kinetic energy of the ship.

(a) If the spaceship weights $10^6 kg$ (without fuel), how much fuel would be needed to accelerate the spaceship to velocity 0.1c, based on energy considerations alone?

(b) Would momentum be conserved in this process?

4. Two protons collide at the Large Hadron Collider (LHC), each traveling at a velocity u such that $\gamma(u) = 7 \cdot 10^3$. A proton weighs about $1\text{GeV}/c^2$.

Note: an electron volt (eV) is the amount of energy it takes to move an electron through a potential difference of 1 Volt = 1 Joule/Coulomb. This means $1eV = 1.6 \times 10^{-19} J$. The electron volt is the standard unit used to describe energies in atomic and particle physics. Masses of subatomic particles are also described by giving their mass energy in electron volts (or $MeV = 10^6 eV$ or $GeV = 10^9 eV$ or larger units). For example, the mass energy of an electron is $m_e c^2 = 0.511 MeV$.

(a) How fast are the protons moving?

(b) The two protons form an unstable particle. What is this particle's mass?

(c) The unstable particle decays to form two hypothetical proton-like particles each with rest mass 1000GeV. How fast are these hypothetical particles going?

(d) Is it possible to produce another hypothetical particle with rest mass $Mc^2 = 16TeV$ in this collision?

Challenge problem. Not to be handed in, but it's a fun one to try. I will post the answer with the solution to the Problem Set.

A spaceship has a drive which exerts a constant force as seen in the spaceship's instantaneous frame of reference. What is this spaceship's position as a function of time?