Physics 200 Problem Set 2

Due at the end of class, Wed Sept 22nd

1. In this question we will see how large relativistic effects are in ordinary life situations.

(a) Prove that

$$\sqrt{1-x} \approx 1 \; - \; \frac{1}{2}x + \text{ terms of order } x^2$$

and

$$\frac{1}{\sqrt{1-x}} \approx 1 \; + \; \frac{1}{2}x + \text{ terms of order } x^2$$

if $x \ll 1$. You will find this fact useful if you let $x = v^2/c^2$.

(b) A frequent flier has flown a million miles. Assuming a typical airplane speed of 700km/h, compute how much younger he is as a result (in seconds).

(c) A supersonic jet is flying at Mach 3 (this means it's traveling at three times the speed of sound, which is 340m/s). The jet is 30 meters long. By how much does it appear shorter to someone on the ground? Compare this with the size of an atom of hydrogen.

2. In year 2075, Fran, an astronaut, is 28 years old. She reads that in 2100 there will be a special mission to the center of the Galaxy, using a new type of drive. She really wants to apply to be part of the mission, but realizes that she will be too old in 2100 to qualify. She decides to sign up for two low-key missions to Alpha Centauri and back. Alpha Centauri is 4.36 light years away from the Earth. Assuming a constant speed, at what fraction of the speed of light does the ship have to travel for Fran to be 10 years younger in 2100 than she would otherwise be? Will she get back in time to go on the 2100 mission?

3. Two trains, A and B, are moving towards each other with relative speed 0.9c. The passing of the two trains (from when their fronts align to when their backs align) takes twice as long as observed from train A as is does as observed from train B. What is the ratio of the length of A to the length of B?

4. The GPS satellites circle the Earth in approximately 12h orbits. At t=0, the clock carried by the satellite and a reference clock on Earth were synchronized. The Earth weights 6×10^{24}kg. After a full orbit, the satellite's clock and the reference clock are compared. Compute, in seconds, the difference in their readings due to special relativistic effects.
Notes: this does not account for the full physical discrepancy, as there is also a general relativistic effect due to gravity being weaker at the satellite's altitude than at sea level. For comparison, national standards agencies maintain clock accuracy of 10^{-9} seconds per day.