Physics 200 Problem Set 11

Due at the end of class, Fri Dec 3rd

Practice problems: do not hand in. Textbook pg 1298-1299, exercises 1, 3, 9, 21, 23, 29, 39, pg 1237 problem 51.

1. Consider a quantum particle of mass m confined by the infinite square well potential to a region of length L. You might want to consult with Tutorial 11 for this question.

(a) Use the uncertainty principle to estimate* the range of velocities we might find if we measure the velocity of the particle in its lowest energy state.

$\star$ Estimate here means 'compute the dependence on L, m and $\hbar$, ignoring overall factors of 2, π, etc ...'

(b) Compare the kinetic energy implied by your estimate in part (a) to the ground state energy and comment on the role the uncertainty principle plays in the phenomenon of zero point energy.

(c) To be more precise, given a particle in the lowest state (normalized) wavefunction

$$\Psi_1 = \begin{cases} \sqrt{\frac{2}{L}}\ \sin\left(\frac{\pi x}{L}\right) & \text{for } 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$

compute Δx, Δp, and compare $\Delta x \Delta p$ to $\hbar/2$. You may use a computer program such as Maple to do the integrals for you.

Consult PS 10 Questions 3 and 4 if you are unsure how to proceed.

To compute the momentum wavefunction, it might help to recall that

$$\sin w = \frac{e^{iw} - e^{-iw}}{2i}$$

2. In Tutorial 11, you showed that the energy eigenstates for the infinite square well are given by

$$\Psi_n(t,x) = \begin{cases} \sqrt{\frac{2}{L}}\ \sin\left(\frac{n\pi x}{L}\right)\ \exp\left(-i\pi n^2 \frac{h}{4mL^2} t\right) & \text{for } 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$

Let $L = 3nm$. Suppose that we have an electron with the wavefunction $\Phi(x,t) = \frac{1}{\sqrt{2}}\left(\Psi_1(x,t) + \Psi_2(x,t)\right)$.

In the Quantum Bound States simulation available here:
http://phet.colorado.edu/en/simulation/bound-states
choose the square potential well and configure it to have the highest available height and width 3nm. This will approximate an infinite square well with that potential. Now configure your wavefunction to Φ and choose to display the Probability Density.

(a) Is Φ a stationary state?

(b) Observe that the evolution of $|\Phi(t, x)|^2$ is periodic. Measure the period using the simulation.

(c) Derive an expression for the period T in terms of L, m and $\hbar$. Substitute numerical values and compare with your result in part (b).

(d) Plot the energy eigenfunctions Ψ_1 and Ψ_2 at $t = 0$ and and $t = T/2$. Use your plots and the concept of constructive and destructive interference to explain why the probability density $|\Phi(x, t)|^2$ moves from left to right and back.

3. In this question we will use the simulation above again. We will study motion of a wavepacket in the Harmonic Oscillator potential.

To make a wavepacket, you will need to choose the Harmonic Oscillator Potential Well. Configure it to have an angular frequency of $2(\text{fs})^{-1}$. Keep the mass at 1m_e. Now, prepare a superposition state by setting your coefficients according to the following formula:

$$c_n = e^{-2} \frac{2^n}{\sqrt{n!}}$$

Watch the time evolution of the probability density. In contrast with the wavepacket for a free particle, the wavepacket we have just defined does not spread out. Instead, it oscillates back and forth like a mass on a spring (which is what the potential represents).

(a) What is the period of oscillations? is it consistent with the angular frequency you set for the potential?

(b) What is the spring constant of this oscillator?

(c) Estimate, by measuring on the screen with a ruler, the amplitude of the oscillations of the center of the wavepacket. When the wavepacket is completely to the left, what is the potential energy of the electron (in eV)?

(d) Now watch the real part of the wavefunction. Can you see how the wavelength changes as the wavepacket oscillates? When is the wavelength shortest?

(e) Allow the wavepacket to reach the bottom of the potential at $x = 0$ and estimate the wavelength by measuring on the screen with a ruler again. Compute the corresponding momentum and kinetic energy (in eV).

(f) Compare your kinetic energy in part (e) with the potential energy in part (c) and comment.

4. In a series of experiments, a gas of exotic molecules containing exactly one electron each is investigated. It is found that:

- When the gas is cold (so all the molecules are in their ground states) and illuminated with light, it absorbs strongly at wavelengths 103.40nm, 137.87nm and 248.16nm.
- when the gas is cold and illuminated with light of short wavelength, the electrons are ejected from the molecules as long as the wavelengths is less than 88.63nm.
- When the gas is heated, it emits light with 6 different wavelengths.

(a) If we chose to set the potential energy outside of the molecule to be zero, how far below zero energy is the ground state of the molecule?

(b) How many discrete energy level does the molecule have?

(c) What wavelengths of light does the heated gas emit?

Hint: Electrons can jump from one energy level to another by emitting/absorbing a photon whose energy is equal to the difference in energies between the two states.